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Out-of-plane QCD radiation in DIS with high p_t jets*

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Abstract: We present a QCD analysis of the cumulative out-of-event-plane momentum distribution in DIS process with emission of high p_t jets. We derive the all-order resummed result to next-to-leading accuracy and estimate the leading power correction. We aim at the same level of accuracy which, in e^+e^- annihilation, seems to be sufficient for making predictions. As is typical of multi-jet observables, the distribution depends on the geometry of the event and the underlying colour structure. This result should provide a powerful method to study QCD dynamics, in particular to constrain the parton distribution functions, to measure the running coupling and to search for genuine non-perturbative effects.

Keywords: QCD, Deep Inelastic Scattering, Jets, Nonperturbative Effects.

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Contents

1. Introduction

The success of the QCD description of e^+e^- event-shape variables makes these observables useful tools to study features of QCD radiation [1], to measure the running coupling [2] and to search for genuine non-perturbative effects [3, 4, 5].

The standard QCD analysis of event shapes involves resummations of all perturbative (PT) terms which are double (DL) and single (SL) logarithmically enhanced and matching with exact fixed order PT results. In addition, to make quantitative predictions, one needs to consider also non-perturbative (NP) power corrections. These standards have been already achieved for a number of 2-jet event shapes ($1-T, C, M^2/Q^2, B$ in e^+e^- annihilation [1, 6, 7] and $1-T, B$ in DIS [8, 9]), i.e. for observables which vanish in the limit of two narrow jets.

Only recently has the attention moved to three-jet shapes in e^+e^- annihilation (thrust minor T_m [10] and D-parameter [11]). These three-jet observables exhibit a rich geometry dependent structure due to the fact that they are sensitive to large angle soft emission (intra-jet radiation). These results have been extended to jet production in hadron collisions [12]. The main difference between processes with or without hadrons in the initial state is that in the first case jet-shape distributions are not collinear and infrared safe (CIS) quantities, but are finite only after factorizing collinear singular contributions from initial state radiation (giving rise to incoming parton distributions at the appropriate hard scale).

In this paper we consider the DIS proton-electron process

$$e+p \to e \;+\; \text{jets} \;+\; \cdots \tag{1.1}$$

in which we select events with high p_t jets with $p_t \sim Q$. The dots represent the initial state jet and intra-jet hadrons. This process involves (at least) three jets: two large p_t jets (generated by two hard partons in the final state recoiling one against the other) and the initial state jet (generated by the incoming parton). For $p_t \sim Q$, the exchanged boson of momentum q, with $Q^2 = -q^2$, can be treated as elementary.

The observable we study is

$$K_{\text{out}} = {\sum_h}' |p_h^{\text{out}}| \,. \tag{1.2}$$

To avoid measurements in the beam region, the sum indicated by $\sum_h'$ extends over all hadrons not in the beam direction. Here p_h^{out} is the out-of-plane momentum of the hadron h with the *event plane* defined as the plane formed by the proton momentum $\vec{P}$ in the Breit frame and the unit vector $\vec{n}$ which enters the definition of thrust major

$$T_M = \max_{\vec{n}} \frac{1}{Q} {\sum_h}' |\vec{p}_h \cdot \vec{n}| \,, \qquad \vec{n}\cdot\vec{P} = 0 \,. \tag{1.3}$$

The jet events we want to select with $p_t \sim Q$ have $T_M = \mathcal{O}(1)$. To select these events we prefer to use, instead of T_M, the $(2+1)$-jet resolution variable y_2 defined by the k_t jet clustering algorithm [13] (see later). (By $(2+1)$-jet we mean two hard outgoing jets in addition to the beam jet.) To avoid small and large T_M we require $y_- < y_2 < y_+$ with $y_\pm$ some fixed values. Using this variable we have fewer hadronization corrections, see [4, 11].

The observable $K_{\rm out}$ is similar to the out-of-plane jet shape studied in e^+e^- annihilation [10] and in Z_0 production in hadron collisions [12]. In the first case the event plane is defined by the thrust and the thrust major axes. In the second case the event plane is fixed by the beam axis and the Z_0 momentum. The reason to analyse distributions in the out-of-plane momentum is that the observable is sufficiently inclusive to allow an analytical study. The analysis of the in-plane momentum components is more involved since one needs to start by fixing the jet rapidities.

Our analysis of $K_{\rm out}$ will make use of the methods introduced for the study of the two observables in [10] and [12]. In the present case we have one hadron in the initial state, so that incoming radiation contributes both to the observable $K_{\rm out}$ and to the parton density evolution. To factorize these two contributions we follow the method used in hadron-hadron collisions [12]. The result is expressed in terms of the following factorized pieces:

- incoming parton densities obtained by resumming all terms $\alpha_{\rm s}^n \ln^n \mu/K_{\rm out}$ (μ is the small factorization scale needed to subtract the collinear singularities and $K_{\rm out}$ is the hard scale for this distribution);

- "radiation factor" characteristic of our observable. Its logarithm is obtained by resumming all DL and SL terms ($\alpha_{\rm s}^n \ln^{n+1} K_{\rm out}/Q$ and $\alpha_{\rm s}^n \ln^n K_{\rm out}/Q$ respectively).

- matching of the resummed result with the fixed order exact calculations [14, 15].

The factorization of the incoming parton densities and the radiation factor is the crucial step for the present analysis. This result is due to coherence and real/virtual cancellations (see [12] and later). As a result, after this factorization procedure, the radiation factor is a CIS quantity similar to the ones entering in the e^+e^- observables.

In order to make quantitative predictions we need to add to the above PT result the $1/Q$-power corrections. We follow the procedure successfully used in the analysis of e^+e^- jet-shape distributions [4]. The definition of the event plane makes our observable sensitive to hard parton recoil. Here only the two final state hard partons can take a recoil, while the initial state one is fixed along the beam axis. This simplifies the treatment both of the PT distribution and of the interplay between PT and NP effects, which is a characteristic feature of all rapidity independent observables.

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The $1/Q$ coefficient is expressed in terms of a single parameter, $\alpha_0(\mu_I)$, given by the integral of the QCD coupling over the region of small momenta $k_t \leq \mu_I$ (the infrared scale μ_I is conventionally chosen to be $\mu_I = 2\,\mathrm{GeV}$, but the results are independent of its specific value). Effects of the non-inclusiveness of K_{out} are included by taking into account the Milan factor $\mathcal{M}$ introduced in [6] and analytically computed in [16]. The NP parameter $\alpha_0(\mu_I)$ is expected to be the same for all jet shape observables linear in the transverse momentum of the emitted hadrons. It has been measured only for 2-jet event shapes and appears to be universal with a reasonable accuracy [5]. It is interesting to investigate if this universality pattern also holds for near-to-planar 3-jet observables.

In order to improve the readability of the paper, in the main text we present only the results and discuss their physical meaning, leaving the detailed derivation of the results to a few technical appendices. In section 2 we define the distribution and specify the phase space region of K_{out} in which we perform the QCD study. In section 3 we describe the PT and NP result for the K_{out} distribution. We stress how the answer has a transparent interpretation based on simple QCD (and kinematical) considerations. In section 4 we improve our theoretical prediction by performing the first-order matching and present some numerical results. Finally, section 5 contains a summary, discussion and conclusions.

2. The process and the observable

We work in the Breit frame

$$q = \frac{Q}{2}(0,0,0,2)\,, \qquad P = \frac{Q}{2x_{\mathrm{B}}}(1,0,0,-1)\,, \qquad x_{\mathrm{B}} = \frac{Q^2}{2(Pq)}\,, \tag{2.1}$$

in which $2x_{\mathrm{B}}P + q$ is at rest. Here P and q are the momenta of the incoming proton and the exchanged vector boson (γ or Z_0).

In this frame, the rapidity (with respect to the direction of the incoming proton) of an emitted hadron with momentum p_h is given by

$$\eta_h = \frac{1}{2}\ln\left(1 + \frac{(qp_h)}{x_{\mathrm{B}}(Pp_h)}\right). \tag{2.2}$$

To avoid measurements in the beam region, the outgoing hadrons p_h are taken in the rapidity range

$$\eta_h < \eta_0 \simeq -\ln\tan\frac{\Theta_0}{2}\,, \tag{2.3}$$

which corresponds to a cut of angle Θ_0 around the beam direction. Similarly, the sums $\sum_h'$ in (1.2) and (1.3) extend over all hadrons with rapidities in the range (2.3).

To select jet events with $p_t \sim Q$ we use the $(2+1)$-jet resolution variable y_2 introduced in the k_t-algorithm for DIS processes [13]. For completeness we recall its definition.

Given the set of all outgoing momenta one defines the "distance" of p_h from the incoming proton momentum

$$y_{hP} = \frac{2}{Q^2}\, E_h^2(1-\cos\theta_{hP})\,. \tag{2.4}$$

For any pair $p_{h'}$ and $p_{h''}$ of outgoing momenta one also defines the "distance"

$$y_{h'h''} = \frac{2}{Q^2}\, \min\{E_{h'}^2, E_{h''}^2\}(1-\cos\theta_{h'h''})\,. \tag{2.5}$$

If y_{hP} is smaller than all $y_{h'h''}$, the hadron p_h is considered part of the beam jet and removed from the outgoing momentum set. Otherwise, the pair of momenta $p_{h'}, p_{h''}$ with the minimum distance $y_{h'h''}$ are substituted with the pseudo-particle (jet) momentum $p_{h'''} = p_{h'} + p_{h''}$ (E-scheme). The procedure is repeated with the new momentum set until only two outgoing momenta p_{h_1}, p_{h_2} are left. Then the final value of y_2 is defined as

$$y_2 \;\equiv\; \min\{y_{h_1P}, y_{h_2P}, y_{h_1h_2}\}\,. \tag{2.6}$$

To select jet events with $p_t \sim Q$, as stated before, we require

$$y_- < y_2 < y_+\,, \tag{2.7}$$

with $y_\pm$ fixed limits.

The distribution we study is then defined as

$$\frac{d\sigma(y_\pm, K_{\rm out})}{dx_{\rm B}\, dQ^2} = \sum_m \int \frac{d\sigma_m}{dx_{\rm B}\, dQ^2}\, \Theta(y_+ - y_2)\Theta(y_2 - y_-)\, \Theta\left(K_{\rm out} - {\sum_{h=1}^{m}}' |p_h^{\rm out}|\right), \tag{2.8}$$

with $d\sigma_m/dx_{\rm B}dQ^2$ the distribution for m emitted hadrons in the process under consideration. Considering the cross section for the y_2-range (2.7)

$$\frac{d\sigma(y_\pm)}{dx_{\rm B}\, dQ^2} = \sum_m \int \frac{d\sigma_m}{dx_{\rm B}\, dQ^2}\, \Theta(y_+ - y_2)\Theta(y_2 - y_-)\,, \tag{2.9}$$

we have the normalized distribution

$$\Sigma(y_\pm, K_{\rm out}) = \frac{d\sigma(y_\pm, K_{\rm out})}{dx_{\rm B}\, dQ^2} \left\{\frac{d\sigma(y_\pm)}{dx_{\rm B}\, dQ^2}\right\}^{-1}. \tag{2.10}$$

Due to the rapidity limitation (2.3) in the definition of y_2, $K_{\rm out}$ and the event plane, the distribution will depend on η_0. To avoid a strong η_0-dependence we will consider η_0 and $K_{\rm out}$ in the range (see later)

$$K_{\rm out} \;\gtrsim\; Q\, e^{-\eta_0}\,. \tag{2.11}$$

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The fact that this distribution is rich in information on the hard process is clear from the fact that it depends not only on the observable $K_{\rm out}$ but also on the variables $y_\pm$ which define the geometry of the jet events with $p_t \sim Q$. In the range (2.11) the PT result does not depend on η_0, to our accuracy. However, as we shall discuss, the power correction depends linearly on η_0. This is due to the fact that the contributions to the observable are uniform in rapidity.

3. QCD result

The QCD calculation of the distribution (2.10) is based on the factorization [17] of parton processes into the following subprocesses:

- elementary hard distribution;
- incoming parton distribution;
- radiation factor corresponding to the observable $K_{\rm out}$.

These factors are described in the following subsections.

3.1 Elementary hard process

For the elementary hard vertex

$$q\,P_1 \to P_2\,P_3\,, \tag{3.1}$$

we introduce the kinematical variables (see appendix A)

$$\xi = \frac{(P_1P_2)}{(P_1q)}\,, \qquad x = \frac{Q^2}{2(P_1q)} > x_{\rm B}\,. \tag{3.2}$$

The invariant masses are

$$Q_{ab}^2 = 2(P_aP_b)\,, \qquad Q_{12}^2 = \frac{\xi}{x}Q^2\,, \qquad Q_{13}^2 = \frac{1-\xi}{x}Q^2\,, \qquad Q_{23}^2 = \frac{1-x}{x}Q^2\,. \tag{3.3}$$

The thrust major for the process (3.1) is given by $T_M = 2\sqrt{\xi(1-\xi)(1-x)/x}$. The substitution $\xi \to (1-\xi)$ interchanges P_2 and P_3, so we distinguish P_2 from P_3 by assuming

$$(P_1P_2)\; < \;(P_1P_3)\,, \tag{3.4}$$

which restricts us to the region $0 < \xi < 1/2$. The variable y_2 for the elementary vertex is given in terms of the variables in (3.2) by

$$y_2 = \begin{cases} \xi^2 + \xi(1-\xi)\dfrac{1-x}{x}\,, & \text{for } x < \dfrac{1+\xi}{1+2\xi}\,, \\[2ex] \left(\dfrac{1-x}{x}\right)\dfrac{x\xi + (1-\xi)(1-x)}{x(1-\xi)+\xi(1-x)}\,, & \text{for } x > \dfrac{1+\xi}{1+2\xi}\,. \end{cases} \tag{3.5}$$

Inverting this one finds $\xi = \xi(x, y_2)$. This function and the relative phase space are discussed in Appendix A.

We consider now the nature of the involved hard primary partons. We identify the incoming parton of momentum P_1 by the index $\tau = q, \bar{q}, g$. Since (3.4) distinguishes P_2 from P_3, in order to completely fix the configurations of the three primary partons, we need to give an additional index $\delta = 1, 2, 3$ identifying the gluon. Therefore the primary partons with momenta $\{P_1, P_2, P_3\}$ are in the following five configurations

$$
\begin{aligned}
\tau = g,\, \delta = 1 \;&\to\; \{gq\bar{q}\}\,, \quad \text{or} \quad \{g\bar{q}q\} \\
\tau = q,\, \delta = 2 \;&\to\; \{qgq\}\,, \\
\tau = \bar{q},\, \delta = 2 \;&\to\; \{\bar{q}g\bar{q}\}\,, \\
\tau = q,\, \delta = 3 \;&\to\; \{qqg\}\,, \\
\tau = \bar{q},\, \delta = 3 \;&\to\; \{\bar{q}\bar{q}g\}\,.
\end{aligned}
\tag{3.6}
$$

In appendix A we give the corresponding five elementary distributions $d\hat{\sigma}_{\tau,\delta,f}$, with f the fermion flavours. The presence of the index τ as well as δ is due to the parity-violating term in the cross-section associated with Z_0 exchange, such that the elementary cross-sections differ for incoming quark and antiquark of the same flavour. If we consider only photon exchange then the index τ is redundant.

3.2 Factorized QCD distribution

The process (1.1) is described in QCD by one incoming parton of momentum p_1 (inside the proton), and two outgoing hard partons p_2, p_3 accompanied by an ensemble of secondary partons k_i

$$q\, p_1 \to p_2\, p_3\, k_1 \cdots k_n\,. \tag{3.7}$$

Taking a small subtraction scale μ (smaller than any other scale in the problem), we assume that p_1 (and the spectators) are parallel to the incoming hadron,

$$p_1 = x_1 P\,. \tag{3.8}$$

Therefore, the observable we study is

$$K_{\text{out}} = |p_{2x}| + |p_{3x}| + {\sum_i}' |k_{ix}|\,, \tag{3.9}$$

where the x-axis corresponds to the out-of-plane direction, the z-axis to the Breit direction, and the event plane is the y-z plane. For large y_2 the hard partons p_2, p_3 are emitted at large angle. For small K_{out}, p_2, p_3 and the secondary parton momenta k_i are near the event plane.

The event plane definition (1.3) corresponds to the condition

$$p_{2x} + \sum_U{}' k_{ix} = p_{3x} + \sum_D{}' k_{ix}\,, \tag{3.10}$$

which, together with momentum conservation, leads directly to

$$p_{2x} = -\sum_U{}' k_{ix} - \frac{1}{2}\sum{}'' k_{ix}\,, \qquad p_{3x} = -\sum_D{}' k_{ix} - \frac{1}{2}\sum{}'' k_{ix}\,. \tag{3.11}$$

Here, by U and D we indicate the up- and down-regions corresponding to partons k_i with $k_{iy} > 0$ and $k_{iy} < 0$ respectively. Again, by $\sum'$ we indicate the sum restricted to secondary partons in the region (2.3). By $\sum''$ we indicate the sum restricted to secondary partons k_i in the "beam-region" with $\eta_i > \eta_0$.

We perform the QCD analysis at the accuracy required to make a quantitative prediction: DL and SL resummation, matching with exact fixed order results, and leading $1/Q$ power correction. The analysis is similar to the one performed in [10, 11, 12]: the starting point is the elementary process (3.1). Then one considers the secondary radiation (soft and/or collinear) in a 3-jet environment. Finally one takes into account the exact matrix element corrections and power corrections.

The application to the present case is described in detail in Appendix B where we show that the distribution can be expressed in the following factorized structure

$$\frac{d\sigma(K_{\rm out}, y_\pm)}{dx_{\rm B}\, dQ^2} = \sum_{\tau,\delta,f} \int_{x_{\rm B}}^{x_M} \frac{dx}{x} \int_{\xi_-}^{\xi_+} d\xi \left(\frac{d\hat{\sigma}_{\tau,\delta,f}}{dx\, d\xi\, dQ^2}\right) \cdot \mathcal{I}_{\tau,\delta,f}(K_{\rm out}, x, \xi, Q, x_{\rm B}, \eta_0)\,, \tag{3.12}$$

where x_M is a function of y_-, the limits $\xi_\pm = \xi_\pm(x, y_\pm)$ select jet events with $p_t \sim Q$ and $d\hat{\sigma}$ are the distributions for the elementary hard process (3.1). See appendix A for the elementary distributions and kinematics.

The distributions $\mathcal{I}$, which resum higher order QCD emission, can be expressed in the following factorized form

$$\mathcal{I}_{\tau,\delta,f} \;=\; C_{\tau,\delta}(\alpha_{\rm s}) \cdot \mathcal{P}_{\tau,f}\left(\frac{x_{\rm B}}{x}, K_{\rm out}\right) \cdot \mathcal{A}_\delta\left(K_{\rm out}, x, \xi, Q, \eta_0\right). \tag{3.13}$$

Here we describe the various factors:

- the factor $\mathcal{P}_{\tau,f}$ is the incoming parton distribution. It is given, for the various cases, by the quark, antiquark or gluon distribution inside the proton

$$\begin{aligned} \mathcal{P}_{q,f}(x, K_{\rm out}) &= q_f(x, K_{\rm out})\,, \quad \mathcal{P}_{\bar{q},f}(x, K_{\rm out}) = \bar{q}_f(x, K_{\rm out})\,, \\ \mathcal{P}_{g,f}(x, K_{\rm out}) &= g(x, K_{\rm out})\,. \end{aligned} \tag{3.14}$$

  We show that here the hard scale is fixed at $K_{\rm out}$ and that the dependence on η_0 can be neglected as long as we take $K_{\rm out}$ sufficiently large in the range (2.11);

- the distribution $\mathcal{A}_\delta$ is the CIS radiation factor which resums powers of $\ln K_{\rm out}/Q$ and is a CIS quantity. It is sensitive only to QCD radiation and therefore does not depend on the flavour (we neglect quark masses). There are various hard scales in $\mathcal{A}_\delta$ (given in terms of the Q_{ab}^2 in (3.3)) which are determined by the SL accuracy analysis. This quantity is similar to 3-jet shape distributions one encounters in e^+e^- annihilation processes [10, 11] and in hadron collisions [12];

- the first factor is the non-logarithmic coefficient function with the expansion

$$C_{\tau,\delta}(\alpha_{\rm s}) = 1 + c_1 \frac{\alpha_{\rm s}}{2\pi} + \mathcal{O}\left(\alpha_{\rm s}^2\right) , \quad \alpha_{\rm s} = \alpha_{\rm s}(Q) . \tag{3.15}$$

  It takes into account hard corrections not included in the other two factors and is obtained from the exact fixed order results.

The factorization of the first two pieces is based on the fact that contributions to $\mathcal{A}$ (to $\mathcal{P}$) come from radiation at angles larger (smaller) than $K_{\rm out}/Q$. This implies that one is able to reconstruct the parton densities $\mathcal{P}$ as for the DIS total cross sections in which one does not analyse the emitted radiation. The only difference here is that the parton density hard scale is given by $K_{\rm out}$ while in the fully inclusive case of DIS total cross section the hard scale is Q. As a result of this factorization, the radiation factor $\mathcal{A}$ can be analysed by the same methods used in e^+e^-.

The basis for the factorized result (3.13) has been discussed in [12] and it is based on coherence and real/virtual cancellations. Since this is a crucial point for our analysis we recall in some detail the relevant steps.

- Since $K_{\rm out}$ is a CIS (global [18]) quantity (its value does not change if one of the emitted particles branches into collinear particles or undergoes soft bremsstrahlung), within SL accuracy we can systematically integrate over the final state collinear branchings.

- Factorizing the phase space and the observable (see (B.4)) by Mellin and Fourier transforms one has that each secondary parton of momentum k_i contributes with an inclusive factor

$$I(k_i) = 1 - \epsilon(k_i) \cdot U(k_i) , \tag{3.16}$$

  where 1 is the virtual term while the factors ϵ and U are the contribution from the real emission. The factor $\epsilon(k_i)$ accounts for the energy loss of the primary incoming parton p_1 due to emission of collinear secondary partons. Therefore, for k_i non-collinear to p_1 we have $\epsilon(k_i) = 1$, while for k_i collinear to p_1 we have $\epsilon(k_i) = z_i^{N-1}$ with $1 - z_i$ the energy fraction of k_i with respect to p_1 and N the usual Mellin moment of the anomalous dimension, see (B.7). The factor $U(k_i)$ depends on the observable and in our case is given by (B.8).

- The crucial point which is the basis of the factorization in (3.13) is that, to SL accuracy for $K_{\rm out} \ll Q$, we can replace (see (B.16))

$$[1-U(k_i)] \simeq \Theta(k_{ti}-k_0)\,, \qquad k_0 \sim K_{\rm out}\,, \tag{3.17}$$

so that we can write

$$I(k_i) \simeq \Theta\left(k_{ti}-k_0\right) + \left[1-z_i^{N-1}\right]\Theta\left(k_0-k_{ti}\right). \tag{3.18}$$

The first term contributes to the radiation factor while the second reconstructs the anomalous dimensions for the various channels. The two contributions do not interfere and, as discussed above, one obtains the factorized expression (3.13).

In the next two subsections we will describe our result for the radiation factor $\mathcal{A}_\delta$. First we describe the PT contribution obtained by resumming the logarithmically enhanced terms at SL accuracy. Then we describe the leading NP corrections originating from the fact that the (virtual) momentum in the argument of the running coupling cannot be prevented from vanishing, even in hard distributions. Matching with the exact first order result is considered later.

3.3 The PT radiation factor and distribution

The PT radiation factor $\mathcal{A}_\delta^{\rm PT}$ can be expressed as (see appendix B)

$$\mathcal{A}_\delta^{\rm PT} \;=\; e^{-R_\delta\left(K_{\rm out}^{-1},x,\xi,Q\right)}\cdot \mathcal{S}_\delta\left(\alpha_{\rm s}\ln\frac{Q}{K_{\rm out}}\right). \tag{3.19}$$

It resums DL and SL contributions originating from the emission of soft or collinear secondary partons k_i in (3.7). The PT radiator R_δ in the first factor contains the DL resummation together with SL contributions coming from the running coupling and the proper hard scales. The second factor is the SL function which resums effects due to multiple radiation, including hard parton recoil. For $K_{\rm out}$ in the region (2.11), this PT result does not depend on η_0. The analysis leading to this result is very similar to that for other 3-jet distributions [10, 11, 12] (see Appendix C). Here we report the relevant expressions and illustrate their physical aspects.

The PT radiator R_δ is given by a sum of three contributions associated with the emission from the three primary partons P_a. For the configuration δ, it is given by

$$R_\delta \;=\; \sum_{a=1}^{3} C_a^{(\delta)}\, r\left(K_{\rm out}^{-1}, \zeta_a^{(\delta)}Q_a^{(\delta)}\right)\,, \tag{3.20}$$

where

$$\begin{aligned}
Q_a^{(\delta)} &= Q_b^{(\delta)} = Q_{ab}\,, &\qquad Q_\delta^{(\delta)} &= \frac{Q_{a\delta}Q_{\delta b}}{Q_{ab}}\,,\\
C_a^{(\delta)} &= C_b^{(\delta)} = C_F\,, & C_\delta^{(\delta)} &= C_A\,,\\
\zeta_a^{(\delta)} &= \zeta_b^{(\delta)} = e^{-\frac{3}{4}}\,, & \zeta_\delta^{(\delta)} &= e^{-\frac{\beta_0}{4N_c}}\,,
\end{aligned} \tag{3.21}$$

with indices a, b corresponding to the fermions and δ to the gluon, i.e. $a \neq b \neq \delta$.

As is typical for a 3-jet quantity, the scale for the quark or antiquark terms of the radiator is the fermion invariant mass, while the scale for the gluon term is given by the gluon transverse momentum with respect to the fermion system. The rescaling factors $\zeta_a^{(\delta)}$ take into account SL contributions from non-soft secondary partons collinear to the primary partons P_a.

In (3.20) r is the following DL function

$$r\left(K_{\rm out}^{-1}, Q'\right) \equiv \int_{K_{\rm out}}^{Q'} \frac{dk}{k} \frac{2\alpha_{\rm s}(2k)}{\pi} \ln \frac{Q'}{2k}\,, \tag{3.22}$$

and k is the out-of-plane component of momentum and $\alpha_{\rm s}$ is taken in the physical scheme [19]. The rescaling factor 2 in the running coupling comes from the integration over the in-plane momentum component. The fact that $K_{\rm out}$ is the lower bound in the PT radiator, to this order, comes from real/virtual cancellation (see (3.17) and (B.16)). The exact expression of the hard scales and the rescaling factors in (3.21) is relevant at SL level.

The factor $\mathcal{S}$ is expressed in terms of the SL function

$$r' = r'(K_{\rm out}^{-1}, Q) \equiv \frac{2\alpha_{\rm s}(K_{\rm out})}{\pi} \ln \frac{Q}{K_{\rm out}}\,, \tag{3.23}$$

given by the logarithmic derivative of r, apart from terms beyond SL accuracy. We find

$$\mathcal{S}_\delta = \frac{e^{-\gamma_E C_T r'}}{\Gamma(1 + C_T r')} \cdot \mathcal{F}\left(C_{12}^{(\delta)} r'\right) \mathcal{F}\left(C_{13}^{(\delta)} r'\right), \qquad C_T = 2C_F + C_A\,, \tag{3.24}$$

where C_T is the total colour charge of the primary partons (see (3.1)). The first factor is the same for all 3-jet shape emission distributions. The second factor depends on the specific observable. The function $\mathcal{F}$ is given by

$$\mathcal{F}(z) = \frac{\Gamma((1+z)/2)}{\sqrt{\pi}\,\Gamma(1+z/2)}\,. \tag{3.25}$$

It takes into account the correct kinematics for the emission and in particular the effect of the recoil of the two outgoing primary partons in (3.1) due to the emission of secondary QCD radiation. Here $C_{1a}^{(\delta)}$ is given by the colour charge of the outgoing parton $a = 2, 3$ plus half of the charge of the incoming parton, in the given configuration,

$$C_{1a}^{(\delta)} = \frac{1}{2} C_1^{(\delta)} + C_a^{(\delta)}\,. \tag{3.26}$$

This combination of colour charges enters due to the kinematics that defines the event plane (see (3.11)), which leads to the vanishing of the sum of the out-of-plane momenta in the "up-region", i.e. with positive y-components, and also in the "down-region". (We are working in the régime in which the effect of the rapidity cut is negligible.) The momenta of outgoing primary partons #2 and #3 are in the up- and down-region respectively. The incoming parton #1 is instead along the Breit axis and emits equally into both regions. Notice that $C_{12}^{(\delta)} + C_{13}^{(\delta)} = C_T$.

Finally the DL and SL resummed PT part of the distribution $\mathcal{I}$ is obtained from (3.13) by using the radiation factor in (3.19)

$$\mathcal{I}^{\rm PT}_{\tau,\delta,f} \;=\; C_{\tau,\delta}(\alpha_{\rm s}) \cdot \mathcal{P}_{\tau,f}\left(\frac{x_{\rm B}}{x}, K_{\rm out}\right) \cdot \mathcal{A}^{\rm PT}_{\delta}\left(K_{\rm out}, x, \xi, Q\right) . \tag{3.27}$$

The coefficient function will be computed at one-loop by using the numerical program DISENT of ref. [14] and subtraction of the one-loop contribution already contained in the two factors $\mathcal{P}$ and $\mathcal{A}$.

To first order in the PT expansion we have

$$\frac{d\mathcal{A}^{\rm PT}_{\delta}}{d\ln K_{\rm out}} = \frac{2\alpha_{\rm s}(Q)}{\pi}\sum_{a=1}^{3} C_a^{(\delta)} \ln\frac{\zeta_a^{(\delta)} Q_a^{(\delta)}}{K_{\rm out}} + \mathcal{O}\left(\alpha_{\rm s}^2\right) . \tag{3.28}$$

The absence of the factor $\frac{1}{2}$ in the logarithm compared with that of equation (3.22) is due to the event-plane kinematics (see (3.11)). For a single gluon k_x (emitted from p_1, p_2 or p_3) only one of the two hard partons p_2 or p_3 takes recoil equal to $|p_{ax}| = |k_x| = K_{\rm out}/2$.

3.4 The distribution including NP corrections

As in other cases of jet-shape distributions, the leading NP correction corresponds to a *shift* in the PT distribution. This is due to the fact that, in the Mellin representation of $\mathcal{I}$, the PT radiator is affected by a NP correction with leading term *linear* in the Mellin variable. Thus its effect corresponds to a shift in the conjugate variable, $K_{\rm out}$, and one has

$$\mathcal{I}_{\tau,\delta,f}(K_{\rm out}) = \mathcal{I}^{\rm PT}_{\tau,\delta,f}\left(K_{\rm out} - \delta K_{\rm out}^{(\delta)}\right) . \tag{3.29}$$

The quantity $\delta K_{\rm out}^{(\delta)}$, which can be cast in the following form

$$\delta K_{\rm out}^{(\delta)} = \lambda^{\rm NP}\left\{\frac{C_{12}^{(\delta)}}{C_T}\cdot\sum_{b=1}^{3} C_b^{(\delta)}\, Y_{2b}^{(\delta)} \;+\; \frac{C_{13}^{(\delta)}}{C_T}\cdot\sum_{b=1}^{3} C_b^{(\delta)}\, Y_{3b}^{(\delta)}\right\}, \tag{3.30}$$

corresponds to the integral over the infrared region of the soft gluon distribution with weight $|k_x|$, the contribution to $K_{\rm out}$ from the very soft gluon generating the NP contribution. The first factor $\lambda^{\rm NP}$ corresponds to the momentum integral (including $|k_x|$ and the running coupling), while $Y_{ab}^{(\delta)}$ is the rapidity interval (more precisely the logarithmic integral of the angle which the very soft gluon forms with the event plane).

The parameter $\lambda^{\rm NP}$, given in (D.6), is expressed in terms of the NP parameter $\alpha_0(\mu_I)$ and the Milan factor $\mathcal{M}$ [6, 16] which takes into account effects of the non-inclusiveness of $K_{\rm out}$. $\lambda^{\rm NP}$ also contains renormalon cancellation terms. The quantity $\alpha_0(\mu_I)$ is the integral of the running coupling in the infrared region

$$\alpha_0(\mu_I) = \int_0^{\mu_I} \frac{dk_t}{\mu_I} \alpha_{\rm s}(k_t)\,. \tag{3.31}$$

The infrared scale μ_I is conventionally chosen to be $\mu_I = 2\,\mathrm{GeV}$, but the results are independent of its specific value. Both $\alpha_0(\mu_I)$ and $\mathcal{M}$ are the same for all jet shape observables linear in the transverse momentum of emitted hadrons.

The derivation of the rapidity intervals $Y_{ab}^{(\delta)}$ is reported in appendix B, see (B.39). The expressions for $b = 1$ are simply given by

$$Y_{21}^{(\delta)} = Y_{31}^{(\delta)} = \ln \frac{x e^{\eta_0} Q_1^{(\delta)}}{Q}\,. \tag{3.32}$$

For the diagonal cases $a = b = 2, 3$ we have the behaviour

$$Y_{22}^{(\delta)} \simeq \ln \frac{2Q_2^{(\delta)}\zeta}{K_{\rm out}} + \mathcal{O}\left(r'\right)\,, \qquad Y_{33}^{(\delta)} \simeq \ln \frac{2Q_3^{(\delta)}\zeta}{K_{\rm out}} + \mathcal{O}\left(r'\right)\,, \tag{3.33}$$

with r' given in (3.23), $Q_b^{(\delta)}$ the hard scales in (3.21) and $\zeta = 2e^{-2}$ the rescaling factor which accounts for the mismatch in the integration over an angle between two vectors and a vector and a plane. The expressions for the off-diagonal cases have more complex behaviours. We have to distinguish two regions. For $\alpha_{\rm s} \ln^2 Q/K_{\rm out} \ll 1$ we have (see (B.42))

$$Y_{23}^{(\delta)} \simeq \frac{\pi}{2\sqrt{C_{13}^{(\delta)}\,\alpha_{\rm s}(Q)}}\,, \qquad Y_{32}^{(\delta)} \simeq \frac{\pi}{2\sqrt{C_{12}^{(\delta)}\,\alpha_{\rm s}(Q)}}\,, \tag{3.34}$$

with corrections of $\mathcal{O}\left(1\right)$. For $\alpha_{\rm s} \ln^2 Q/K_{\rm out} \gg 1$, we have (see (B.41))

$$Y_{32}^{(\delta)} \simeq \ln \frac{2Q_2^{(\delta)}\zeta}{K_{\rm out}} + \mathcal{O}\left(r'\right)\,, \qquad Y_{23}^{(\delta)} \simeq \ln \frac{2Q_3^{(\delta)}\zeta}{K_{\rm out}} + \mathcal{O}\left(r'\right)\,. \tag{3.35}$$

Before illustrating and discussing these expressions, we observe the following features of the distribution obtained in (3.29). Since $\delta K_{\rm out}^{(\delta)}$ depends on $K_{\rm out}$, the PT distribution is actually deformed. The dependence on η_0, the available rapidity interval for the measurement, enters only through the NP shift (the PT distribution is independent of η_0, to our accuracy, as long as we take $K_{\rm out}$ in the region (2.11)). The dependence of $\delta K_{\rm out}^{(\delta)}$ on y_2 (through $Q_b^{(\delta)}$), implies that the deformation of the PT distribution differs for different geometries of the process. To this order (leading $1/Q$ power) we can neglect NP corrections arising from the anomalous dimension [4] which are of subleading order $1/Q^2$. Finally, we observe that the presence of $\ln K_{\rm out}$

and $1/\sqrt{\alpha_s}$ contributions to the shift are features common to the case of rapidity independent observables such as, broadening [7], T_m [10] or the out-of-plane momentum in hadronic collisions studied in [12].

We now discuss how the above expressions for the rapidity intervals $Y_{ab}^{(\delta)}$ are based on simple QCD considerations and on the kinematics of the event-plane (see (3.11)). In the following we focus on the term (3.30)

$$\frac{C_{12}^{(\delta)}}{C_T} \cdot C_b^{(\delta)} \cdot Y_{2b}^{(\delta)}\,, \tag{3.36}$$

the other is obtain by exchanging $2 \leftrightarrow 3$. It corresponds to the contribution in which a PT gluon is emitted in the up-region (this happens with probability $C_{12}^{(\delta)}/C_T$) and a very soft gluon, generating the NP correction, is emitted off the primary parton $b = 1,2,3$ (with colour factor $C_b^{(\delta)}$). For this contribution we consider the various cases.

Case $b = 1$. The incoming parton p_1, emitting the very soft gluon, is never affected by recoil since it is aligned along the Breit axis. Therefore the rapidity interval Y_{21} is fixed by the boundary η_0, the available rapidity interval for the measurement (see (2.3)). The remaining term $\ln xQ_1^{(\delta)}/Q$ is related to the standard parton #1 hard scale and a boost from the Breit frame of full process to the hard elementary vertex, see (D.10);

Case $b = 2$. The outgoing parton p_2 and the PT gluon are in the same region (the up-region) so that, from the event plane kinematics (3.11), p_2 undergoes recoil with $|p_{2x}| = K_{\rm out}/2$. Since $\ln(\zeta Q_2^{(\delta)}/|p_{2x}|)$ fixes the rapidity interval for the very soft gluon emitted by p_2, we obtain the result (3.33);

Case $b = 3$. Here we cannot be limited to considering the contribution from a single PT gluon emitted in the up-region since, from the event plane kinematics (3.11), one would have $p_{3x} = 0$ and the rapidity interval Y_{23} would diverge (for a zero mass gluon). We then need to consider high-order PT contributions which push parton #3 off the plane by a certain p_{3x}. The distribution in the recoil momentum p_{3x} has two different régimes:

- for $\alpha_s \ln^2 Q/K_{\rm out} \ll 1$ there are few secondary partons and the PT distribution is given by Sudakov form factors. Integrating $\ln|p_{3x}|$, the rapidity interval contribution, we obtain the behaviour in (3.34) (here $C_{13}^{(\delta)}$ is the coefficient of the DL exponent of the $|p_{3x}|$ Sudakov factor);

- for $\alpha_s \ln^2 Q/K_{\rm out} \gg 1$ the PT radiation is well developed, so that p_3 takes recoil with $|p_{3x}| \sim K_{\rm out}$ and we obtain the behaviour (3.35).

4. Matching and numerical analysis

The final step needed to obtain a quantitative prediction is the calculation of the non-logarithmic coefficient function (3.15) from the exact matrix element results. We use the numerical program DISENT of ref. [14], which includes only the γ exchanged contribution, and we compute the first order coefficient c_1 by subtracting the DL and SL terms already contained in the PT resummed result. The Z_0 contribution will be important at large values of Q. For example, for $Q = 20\,\mathrm{GeV}$ the Z_0 contribution to the cross section is of order 0.5% (see (A.8)).

Taking into account only γ exchange simplifies considerably the expressions since we only need δ to identify the configuration, while τ becomes redundant. The distribution (2.8) and the cross-section for the high p_t jet (2.9) are given by

$$\frac{d\sigma^{(\gamma)}(y_\pm, K_{\mathrm{out}})}{dx_{\mathrm{B}}\, dQ^2} = \sum_{\delta,f} \int_{x_{\mathrm{B}}}^{x_M} \frac{dx}{x} \int_{\xi_-}^{\xi_+} d\xi \left(\frac{d\hat{\sigma}^{(\gamma)}_{\delta,f}}{dx\, d\xi\, dQ^2} \right) \cdot \mathcal{I}^{(\gamma)}_{\delta,f}(K_{\mathrm{out}}, x, \xi,\, Q, x_{\mathrm{B}}, \eta_0)\,,$$

$$\frac{d\sigma^{(\gamma)}(y_\pm)}{dx_{\mathrm{B}}\, dQ^2} = \sum_{\delta,f} \int_{x_{\mathrm{B}}}^{x_M} \frac{dx}{x} \int_{\xi_-}^{\xi_+} d\xi \left(\frac{d\hat{\sigma}^{(\gamma)}_{\delta,f}}{dx\, d\xi\, dQ^2} \right) \cdot \mathcal{P}^{(\gamma)}_{\delta,f}\left(\frac{x_{\mathrm{B}}}{x}, Q\right). \tag{4.1}$$

where x_M and $\xi_\pm = \xi_\pm(x, y_\pm)$ are found in appendix A. The hard elementary distribution is given by (see appendix A)

$$\begin{aligned}
\frac{d\hat{\sigma}^{(\gamma)}_{\delta,f}}{dx\, d\xi\, dQ^2} &= \frac{\alpha^2 \alpha_{\mathrm{s}}}{Q^4} e_f^2 \left[(2-2y+y^2)\; C_T^\delta(x,\xi) + 2(1-y)\, C_L^\delta(x,\xi) \right]\,,\\
C_T^3 &= C_F \left[\frac{x^2+\xi^2}{(1-x)(1-\xi)} + 2(1+x\xi) \right]\,, \quad C_T^1 = \left[x^2 + (1-x)^2 \right] \frac{\xi^2 + (1-\xi)^2}{\xi(1-\xi)}\,,\\
C_L^3 &= C_F \cdot 4x\xi\,, \qquad C_L^1 = 8x(1-x)\,, \qquad C^2_{T/L}(x,\xi) = C^3_{T/L}(x, 1-\xi)\,,
\end{aligned} \tag{4.2}$$

where $y = Q^2/(x_{\mathrm{B}} s)$ with $\sqrt{s}$ the centre of mass energy of the process.

We start with the resummed PT result $\mathcal{I}^{\mathrm{PT}}$ (3.27) which in this case takes the form

$$\mathcal{I}^{(\gamma)\,\mathrm{PT}}_{\delta,f} = C^{(\gamma)}_\delta(\alpha_{\mathrm{s}}) \cdot \mathcal{P}^{(\gamma)}_{\delta,f}\left(\frac{x_{\mathrm{B}}}{x}, K_{\mathrm{out}}\right) \cdot \mathcal{A}^{\mathrm{PT}}_\delta\left(K_{\mathrm{out}}, x, \xi,\, Q, x_{\mathrm{B}}\right), \tag{4.3}$$

with the incoming parton distribution $\mathcal{P}^{(\gamma)}_\delta$ given by

$$\mathcal{P}^{(\gamma)}_{\delta,f}(x, K_{\mathrm{out}}) = \begin{cases} g(x, K_{\mathrm{out}})\,, & \delta = 1 \\ q_f(x, K_{\mathrm{out}}) + \bar{q}_f(x, K_{\mathrm{out}})\,, & \delta = 2, 3\,. \end{cases} \tag{4.4}$$

The PT radiation factor $\mathcal{A}^{\mathrm{PT}}_\delta$ is the same whether we include the Z_0 or not, and is given in (3.19).

4.1 Matching

We now discuss the first order matching. To determine $c_1^{(\gamma)}$, the first order term of the coefficient function $C_\delta^{(\gamma)}$ (see (3.15)), we expand the normalized resummed distribution to one loop

$$\begin{aligned}
\Sigma^{(\gamma)}(y_\pm, K_{\mathrm{out}}) &= \frac{d\sigma^{(\gamma)}(y_\pm, K_{\mathrm{out}})}{dx_{\mathrm{B}}\, dQ^2} \left\{ \frac{d\sigma^{(\gamma)}(y_\pm)}{dx_{\mathrm{B}}\, dQ^2} \right\}^{-1} \\
&= 1 + \frac{\alpha_{\mathrm{s}}(Q)}{2\pi} \left(G_{12} \ln^2 \frac{Q}{K_{\mathrm{out}}} + G_{11}^{(\gamma)}(y_\pm) \ln \frac{Q}{K_{\mathrm{out}}} + c_1^{(\gamma)}(y_\pm, K_{\mathrm{out}}) \right) + \cdots
\end{aligned} \tag{4.5}$$

and compare with the result provided by the numerical program DISENT [14]. From the resummed result we have $G_{12} = -2C_T$ and

$$G_{11}^{(\gamma)}(y_\pm)\,\frac{d\sigma^{(\gamma)}(y_\pm)}{dx_{\rm B}\,dQ^2} = \sum_{\delta,f}\int_{x_{\rm B}}^{x_M}\frac{dx}{x}\int_{\xi_-}^{\xi_+}d\xi\left(\frac{d\hat\sigma_{\delta,f}^{(\gamma)}}{dx\,d\xi\,dQ^2}\right)\cdot \\ \cdot\left\{-4\sum_a C_a^{(\delta)}\,\ln\frac{\zeta_a^{(\delta)}Q_a^{(\delta)}}{Q} - \frac{2\pi}{\alpha_{\rm s}(Q)}\frac{\partial}{\partial\ln Q}\right\}\cdot\mathcal{P}_{\delta,f}^{(\gamma)}\left(\frac{x_{\rm B}}{x},Q\right). \tag{4.6}$$

We have verified that the above coefficients G_{12} and $G_{11}^{(\gamma)}$ are correctly reproduced by DISENT and by subtraction we deduce the non-logarithmic one loop coefficient $c_1^{(\gamma)}(y_\pm, K_{\rm out})$.

We use the first-order (Log R)-matching prescription and write

$$C_\delta^{(\gamma)}(\alpha_{\rm s})\Big|_{\rm mat} = e^{\frac{\alpha_{\rm S}}{2\pi}c_1(y_\pm,K_{\rm out})}\,, \tag{4.7}$$

with $\alpha_{\rm s} = \alpha_{\overline{\rm MS}}(Q)$. We also implement the correct kinematical bound for $K_{\rm out} < K_{\rm out}^M$ (with $K_{\rm out}^M$ obtained from DISENT) by replacing $K_{\rm out}$ by $K'_{\rm out}$, implicitly defined by

$$\frac{Q}{K_{\rm out}} \to \frac{Q}{K'_{\rm out}} = \frac{Q}{K_{\rm out}} - \frac{Q}{K_{\rm out}^M} + 1\,, \tag{4.8}$$

in the final PT result (4.3) so that the differential distribution vanishes for $K_{\rm out} = K_{\rm out}^M$. Finally we make the replacement (3.29), (3.30) in order to include the NP correction.

4.2 Numerical results

To summarize, the QCD prediction for the normalized distribution $\Sigma(y_\pm, K_{\rm out})$ in (2.10) is obtained (for exchanged γ) from the ratio of the two distributions in (4.1) in which the integrand $\mathcal{I}$ is given by (4.3) with the coefficient function given in (4.7) and using the substitution (4.8) of $K_{\rm out}$. The radiation factor $\mathcal{A}$ is given, at PT level, by (3.19). Leading power corrections are included by performing the substitutions in (3.29) and (3.30).

We now report some plots for $d\Sigma(y_\pm, K_{\rm out})/dK_{\rm out}$ (data are not yet available) for $s = 98400\,{\rm GeV}^2$ and the rapidity cut $\eta_0 = 3$. A smaller rapidity cut, i.e. a larger value of η_0, would have two advantages: the PT resummation is valid for smaller $K_{\rm out}$, and the NP shift becomes larger; but this needs to be balanced against the experimental resolution and the need to exclude the proton remnant.

The QCD predictions are all obtained for the following choices: the parton density functions in [20], set MRST2001; the NP parameter $\alpha_0(\mu_I) = 0.52$, a value in the range determined by the analysis of 2-jet observables in e^+e^- annihilation [21] and the running coupling $\alpha_{\rm s}(M_Z) = 0.119$.

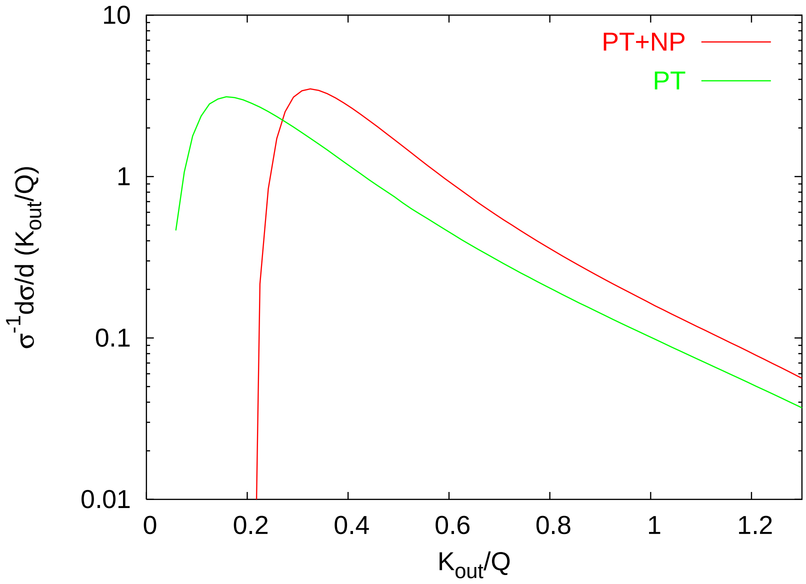

Figure 1: QCD prediction for $x_{\rm B} = 0.1$, $Q^2 = 400\,{\rm GeV}^2$ and $y_- = 0.1$. In the NP correction we have taken $\alpha_0(2\,{\rm GeV}) = 0.52$.

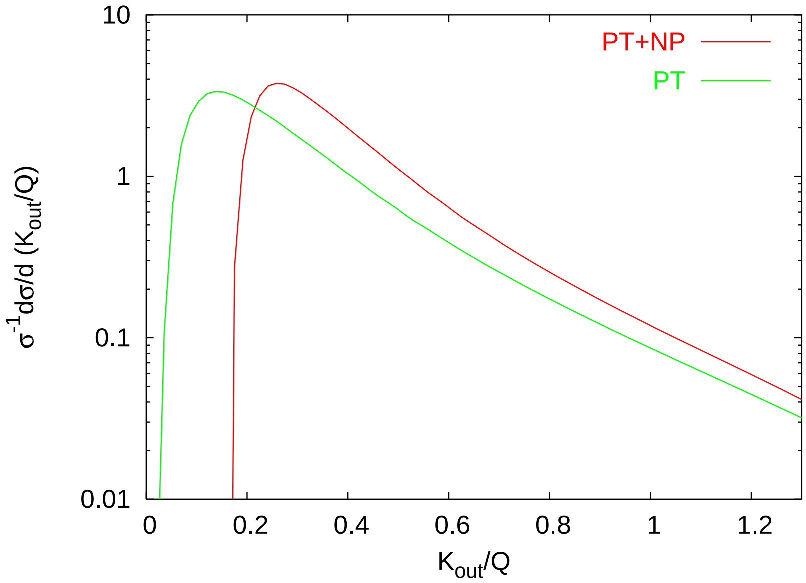

Figure 2: The same as figure 1 but with $Q^2 = 900\,{\rm GeV}^2$.

Figures 1 and 2 show the differential $K_{\rm out}$ distribution for $Q = 20\,{\rm GeV}$ and $Q = 30\,{\rm GeV}$, and $x_{\rm B} = 0.1$. In order to prevent T_M from being too large it is enough to let y_+ go to its maximum kinematically allowed value $y_+ = 2.5$ (see (A.4) and (A.5)). From these pictures one notices the $1/Q$ dependence of the power correction. One can also observe that the shift is larger at smaller $K_{\rm out}$, so that the distribution is squeezed. This is due to the $\ln K_{\rm out}$ dependence of the NP correction (see discussion in section 3.4).

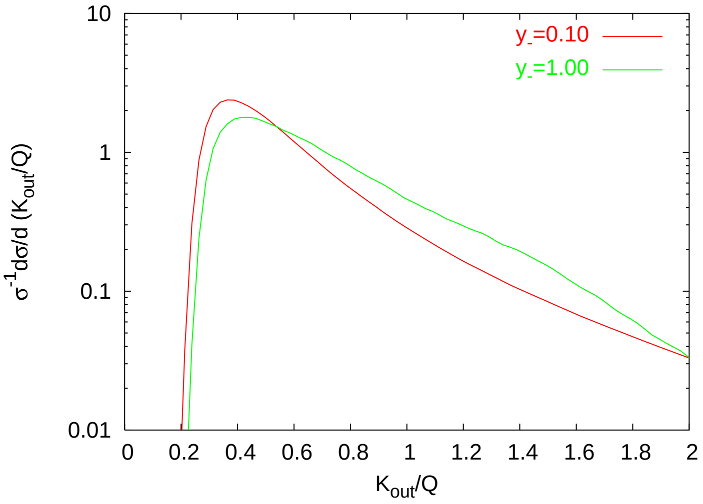

Figure 3: The full (PT+NP) differential K_{out} distribution for two different values of y_-, for fixed $y_+ = 2.5$, $x_{\mathrm{B}} = 0.01$ and $Q^2 = 400\,\mathrm{GeV}^2$.

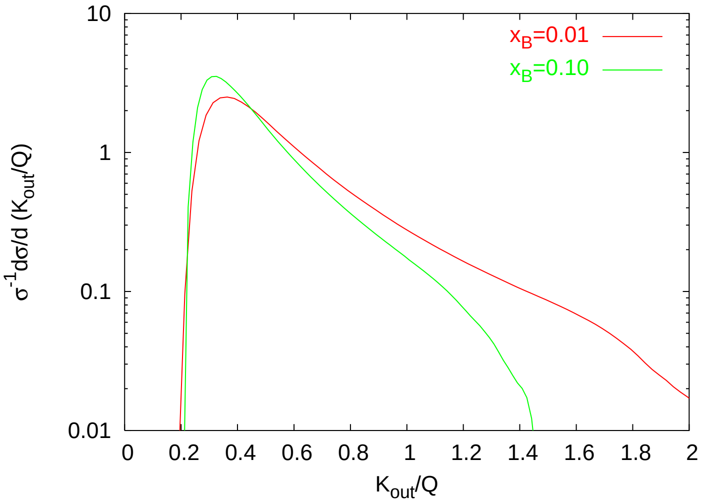

Figure 4: The full (PT+NP) differential K_{out} distribution for two different values of x_{B}, for $y_- = 0.05$, $y_+ = 2.5$, and $Q^2 = 400\,\mathrm{GeV}^2$.

The fact that our predictions are also geometry dependent can be seen from figure 3, where we plot the PT+NP curve for two different values of y_-, fixing $y_+ = 2.5$.

Finally, figure 4 shows how the distribution depends on the value of x_{B}. The distribution dies faster by increasing x_{B}. Indeed, for increasing x_{B} the centre-of-mass energy of the hard system decreases, so that the phase space for producing hadrons with large out-of-plane momentum is reduced. Moreover, by increasing x_{B} the configuration with incoming quark or antiquark ($\delta = 2, 3$) becomes more probable.

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5. Discussion and conclusion

In this paper we have extended our knowledge of three-jet physics to the case of a near-to-planar observable in hard electron-proton scattering, defined analogously to the thrust-minor in e^+e^- annihilation. On the PT side, this observable exhibits a rich colour and geometry dependence. Large angle gluon radiation contributes in setting the relevant scales for the PT radiator (see (3.21)). The radiator itself is a sum of three contributions, one for each emitting parton, each one characterized by a different hard scale: for a quark or antiquark it is the invariant mass of the fermion system, for a gluon it is its (invariant) transverse momentum with respect to the fermion pair. Such a structure, due to the universality of soft radiation, is found to be common to all near-to-planar 3-jet shape variables encountered so far (see [10, 11, 12]).

Since the observable $K_{\rm out}$ is uniform in rapidity, hard parton recoil contributes both to the definition of the event plane and to the observable. It contributes then to the PT distribution at SL level.

The thrust major axis is determined only by the hard parton system and cannot be changed (at least at SL accuracy) by secondary soft or collinear parton radiation. This implies that there exists no correlation between the up- and down-regions (see the definition of the event-plane (3.11)). This property makes the analysis of our observable much simpler than that of the thrust minor T_m distribution in e^+e^-. In particular, we are able to write a closed formula for the SL function $\mathcal{S}_\delta$ in (3.24) which embodies the contributions of hard parton recoil.

Our observable exhibits $1/Q$ power corrections arising from the running of the coupling into the infrared region. For $K_{\rm out} \gg \Lambda_{\rm QCD}$ they can be taken into account as a shift of the PT distribution (see (3.29)). The shift $\delta K_{\rm out}^{(\delta)}$ depends logarithmically on the observable itself, so that the effect of the shift is actually a deformation of the PT distribution. Since $K_{\rm out}$ accumulates contributions from partons of any rapidity, one has to carefully consider effective rapidity cut-off. Along the beam axis the rapidity is bounded by the experimental resolution η_0. Along the direction of the two outgoing hard partons, it is their displacement from the event plane which provides an effective rapidity cut-off. Averaging such a NP correction over the hard parton PT recoil distribution gives rise to contributions of order $\ln K_{\rm out}$ or $1/\sqrt{\alpha_{\rm s}}$ according to the $K_{\rm out}$ régimes. The form of the power corrections can be simply interpreted on the basis of the event plane kinematics (see (3.11)), as explained in detail in section 3.

As in other 3-jet observables, the magnitude of the NP shift is expected to be roughly twice as large as the corresponding one for 2-jet shape variables (the characteristic weight $2C_F$ of the DL contribution of the logarithm of 2-jet distribution becomes here a $2C_F + C_A$ weight). This implies that higher order power corrections may become important, see [22]. The comparison with data (not yet available) will be needed to check the validity of our method.

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A. Kinematics and elementary partonic cross-sections

For the elementary process (3.1) we may write the parton momenta P_1, P_2 as

$$\begin{aligned} P_1 &= \frac{Q}{2x}(1,0,0,-1)\,, \qquad & P_2 &= \frac{Q}{2}(z_0,0,T_M,z_3)\,, \\ z_0 &= \xi + \frac{(1-\xi)(1-x)}{x}\,, \qquad & z_3 &= \xi - \frac{(1-\xi)(1-x)}{x}\,, \end{aligned} \tag{A.1}$$

in terms of the variables in (3.2). Distinguishing P_2 and P_3 according to (3.4), the variable ξ is given in terms of T_M by

$$\xi = \frac{1}{2}\left[1 - \sqrt{1 - \frac{xT_M^2}{1-x}}\right], \tag{A.2}$$

and in terms of y_2 by the inverse of (3.5), which is

$$\begin{aligned} \xi &= \frac{1-x}{2(2x-1)}\left[\sqrt{1+\frac{4x(2x-1)y_2}{(1-x)^2}} - 1\right], \quad & &\begin{cases} 0 < x < \frac{3}{4}, & 0 < y_2 < \frac{1}{4x} \\ \frac{3}{4} < x < 1, & 0 < y_2 < \frac{2(1-x)^2}{x(2x-1)} \end{cases} \\ \xi &= \frac{x^2y_2 - (1-x)^2}{(2x-1)(1-x+xy_2)}\,, & &\frac{3}{4} < x < 1, \quad \frac{2(1-x)^2}{x(2x-1)} < y_2 < \frac{1-x}{x} \end{aligned} \tag{A.3}$$

Note that at any fixed value of y_2, the freedom to vary x results in events with a range of T_M values. If $y_2 < \frac{1}{3}$, the kinematic upper limit on x is $x = \frac{1}{1+y_2}$, at which $\xi = \frac{1}{2}$ and the lower bound on T_M is therefore $\sqrt{y_2}$. The lower limit on x is the Bjorken variable $x_{\rm B}$, at which $\xi = \xi(x_{\rm B}, y_2) < x_{\rm B} y_2$. (This is obtained by expanding (A.3) about $x_{\rm B} = 0$: the difference from equality is of order $x_{\rm B}^2$.) Consequently, $T_M < 2\sqrt{y_2}$. Therefore, restricting ourselves to events in the range (2.7), with $y_- < \frac{1}{3} < y_+$, leads to a selection of events whose values of T_M lie in the range

$$\sqrt{y_-} < T_M < 2\sqrt{y_+}\,. \tag{A.4}$$

This of course does not select all events with T_M in this range; it is merely a means of selecting events whose T_M does not lie outside it, so that we do not need to consider logarithms of T_M. The phase-space in terms of the variables (x, ξ) then becomes:

$$
\begin{aligned}
& x_{\rm B} < x < x_M\,, \quad x_M = \begin{cases} \dfrac{1}{1+y_-}\,, & y_- < 1/3 \\ \dfrac{1}{4y_-}\,, & y_- > 1/3 \end{cases}\,, \\
& \xi_- < \xi < \xi_+\,, \quad \xi_- = \xi(x, y_-)\,, \quad \xi_+ = \begin{cases} \xi(x, y_+)\,, & x < \dfrac{1}{4y_+} \\ \frac{1}{2}\,, & x > \dfrac{1}{4y_+} \end{cases}\,.
\end{aligned} \tag{A.5}
$$

Since we consider only diagrams with the exchange of a single photon or Z_0, the individual partonic cross-sections may be decomposed according to:

$$
\begin{aligned}
\frac{d\hat{\sigma}_{\tau,\delta,f}}{dx\, d\xi\, dQ^2} = \frac{\alpha^2 \alpha_{\rm s}}{Q^4} \Big\{ & C^f(Q^2) \left[\left(2 - 2y + y^2\right) C_T^{\tau,\delta}(x, \xi) + 2(1-y) C_L^{\tau,\delta}(x, \xi) \right] + \\
& + D^f(Q^2) y(2-y) C_3^{\tau,\delta}(x, \xi) \Big\}\,,
\end{aligned} \tag{A.6}
$$

into transverse, longitudinal and parity-violating terms. Here y is defined by

$$
y = \frac{(Pq)}{(PP_e)} = \frac{Q^2}{x_{\rm B} s}\,, \tag{A.7}
$$

where P_e is the momentum of the incident electron, and s is the centre-of-mass energy squared of the collision (we neglect the proton mass). The flavour-dependent functions $C^f(Q^2)$ and $D^f(Q^2)$ show how the γ and Z exchange diagrams combine:

$$
\begin{aligned}
C^f(Q^2) &= e_f^2 - \frac{2 e_f V_f V_e}{\sin^2 2\theta_W} \left(\frac{Q^2}{Q^2 + M^2} \right) + \frac{(V_f^2 + A_f^2)(V_e^2 + A_e^2)}{\sin^4 2\theta_W} \left(\frac{Q^2}{Q^2 + M^2} \right)^2\,, \\
D^f(Q^2) &= - \frac{2 e_f A_q A_e}{\sin^2 2\theta_W} \left(\frac{Q^2}{Q^2 + M^2} \right) + \frac{4 V_f A_f V_e A_e}{\sin^4 2\theta_W} \left(\frac{Q^2}{Q^2 + M^2} \right)^2\,.
\end{aligned} \tag{A.8}
$$

The functions $C_{T/L/3}^{\tau,\delta}(x, \xi)$ are the one loop QCD elementary square matrix elements for the hard vertex (there is no contribution of order $\mathcal{O}\left(\alpha_{\rm s}^0\right)$ since we require transverse jets). We have [23]

$$
\begin{aligned}
C_T^{q,3} &= C_F \left[\frac{x^2 + \xi^2}{(1-x)(1-\xi)} + 2(1 + x\xi) \right]\,, \\
C_T^{g,1} &= \left[x^2 + (1-x)^2 \right] \frac{\xi^2 + (1-\xi)^2}{\xi(1-\xi)}\,, \\
C_L^{q,3} &= C_F \cdot 4x\xi\,, \qquad C_L^{g,1} = 8x(1-x)\,, \\
C_3^{q,3} &= C_F \left[\frac{x^2 + \xi^2}{(1-x)(1-\xi)} + 2(x + \xi) \right]\,, \qquad C_3^{g,1} = 0\,, \\
C_{T/L/3}^{q,2}(x, \xi) &= C_{T/L/3}^{q,3}(x, 1-\xi)\,,\ C_{T/L}^{\bar{q},\delta}(x, \xi) = C_{T/L}^{q,\delta}(x, \xi)\,,\ C_3^{\bar{q},\delta}(x, \xi) = -C_3^{q,\delta}(x, \xi)\,.
\end{aligned}
$$

B. Resumming QCD emission

The QCD results illustrated in section 3 are based on calculations similar to those performed for other 3-jet shape distributions in e^+e^- annihilation [10, 11] and in hadronic collisions [12]. Here we follow the method and use the results developed and obtained there. We do not report the full details of the calculations but simply discuss the main features characteristic of the present distribution: for details refer to the later appendices and to the previous papers. In particular here we

- deduce, in the present context, the factorized structure in (3.13) and identify the hard scale entering the incoming parton distribution $\mathcal{P}$;
- obtain the PT radiation factor at the SL accuracy (3.19);
- compute the leading order NP corrections giving rise to the shift (3.29);
- show that, for $K_{\rm out}$ in the range (2.11), the η_0 dependence enters only in the NP shift.

B.1 Resummation of the distribution

Considering the region $K_{\rm out} \ll Q$, the starting point is the factorization of the square amplitude for the emission of n secondary soft partons in the process (3.7) (contributions from hard collinear secondary emission are included later). We have

$$|M_{\tau,\delta,f,n}(k_1 \dots k_n)|^2 \simeq |M_{\tau,\delta,f,0}|^2 \cdot S_{\delta,n}(k_1 \dots k_n)\,. \tag{B.1}$$

The first factor is the Born squared amplitude which gives rise to the elementary hard distribution $d\hat{\sigma}_{\tau,\delta,f}$ in (3.12). The second factor is the distribution in the soft partons emitted from the system of the three hard partons p_1, p_2 and p_3 in (3.7). It depends on the colour charges of the emitters which are identified by the configurations $\delta = 1, 2, 3$, the index of the primary gluon momentum. Since soft radiation is universal and does not change the nature of incoming parton, $S_{\delta,n}$ does not depend on τ and f. For soft emission, the primary partons p_a differ from the hard primary parton momenta P_a (depending on Q, x, ξ, see Appendix A) by small recoil components.

By using (B.1), the soft contributions to the cross-section are resummed by

$$\begin{aligned}\frac{d\sigma(K_{\rm out}, y_\pm)}{dx_{\rm B}\, dQ^2} = \sum_{\tau,\delta,f} \int_{x_{\rm B}}^{x_M} dx \int_{\xi_-}^{\xi_+} d\xi \left(\frac{d\hat{\sigma}_{\tau,\delta,f}}{dx\, d\xi\, dQ^2}\right) \int_0^1 dx_1 \mathcal{P}_{\tau,f}\,(x_1,\mu) \,\times \\ \times \sum_n \frac{1}{n!} \prod_{i=1}^n \int \frac{d^3k_i}{\pi\omega_i} \int d\Phi_n \cdot S_{\delta,n}\,,\end{aligned} \tag{B.2}$$

with x_M given in appendix A and x_1 the momentum fraction (3.8) of the incoming parton and μ the (small) subtraction scale. The phase space $d\Phi_n$ fixes the observable

$K_{\rm out}$, the event plane and x, the Bjorken variable for the hard elementary distribution $d\hat{\sigma}$. The momentum fraction of the parton entering the hard scattering is then

$$\frac{x_{\rm B}}{x} = x_1 \prod_{i \in \mathcal{C}_1} z_i \,, \tag{B.3}$$

where z_i are the collinear splitting fractions for radiation of secondary partons k_i in the region $\mathcal{C}_1$ collinear to p_1. The phase space $d\Phi_n$ is then given by

$$\begin{aligned} d\Phi_n = dp_{2x}\, dp_{3x}\, \Theta\left(K_{\rm out} - |p_{2x}| - |p_{3x}| - {\sum_i}' |k_{ix}|\right) \delta\left(x_{\rm B} - x x_1 \prod_{i\in\mathcal{C}_1} z_i\right) \times \\ \times \delta\left(p_{2x} + {\sum_{i\in U}}' k_{ix} + \frac{1}{2}{\sum_i}'' k_{ix}\right) \delta\left(p_{3x} + {\sum_{i\in D}}' k_{ix} + \frac{1}{2}{\sum_i}'' k_{ix}\right), \end{aligned} \tag{B.4}$$

where the last two delta functions fix the event plane, as shown in (3.11).

So the radiation factor $\mathcal{I}$ in (3.12) takes the form

$$\mathcal{I}_{\tau,\delta,f} = x \int_0^1 dx_1 \mathcal{P}_{\tau,f}\,(x_1,\mu) \sum_n \frac{1}{n!} \prod_{i=1}^n \int \frac{d^3 k_i}{\pi \omega_i} \int d\Phi_n \cdot S_{\delta,n} \,. \tag{B.5}$$

To resum the secondary parton emissions we use the factorization structure of the soft emission factor $S_{\delta,n}$. The phase space $d\Phi_n$ can be factorized by Mellin and Fourier transforms. We get

$$\begin{aligned} \int d\Phi_n = \frac{1}{x_{\rm B}} \int \frac{dN}{2\pi i} \left(\frac{x x_1}{x_{\rm B}}\right)^{N-1} \int \frac{d\nu\, e^{\nu K_{\rm out}}}{2\pi i \nu} \times \\ \times \prod_{a=2,3} \int_{-\infty}^{\infty} \frac{\nu d\beta_a dp_{ax}}{2\pi} e^{-\nu(|p_{ax}| - i\beta_a p_{ax})} \prod_i \epsilon(z_i)\, U(k_i) \,, \end{aligned} \tag{B.6}$$

where

$$\begin{cases} \epsilon(z) = z^{N-1} \,, & \text{for} \quad k \in \mathcal{C}_1 \,, \\ \epsilon(z) = 1 \,, & \text{for} \quad k \notin \mathcal{C}_1 \,, \end{cases} \tag{B.7}$$

and

$$\begin{cases} U(k) = u_2(k_x) & \equiv e^{-\nu(|k_x| - i\beta_2 k_x)} \quad \text{for } k_y > 0 \,,\ \eta_k < \eta_0 \,, \\ U(k) = u_3(k_x) & \equiv e^{-\nu(|k_x| - i\beta_3 k_x)} \quad \text{for } k_y < 0 \,,\ \eta_k < \eta_0 \,, \\ U(k) = u_0(k_x) & \equiv e^{i\frac{\nu}{2}(\beta_2 + \beta_3) k_x} \qquad \text{for } \eta_k > \eta_0 \,. \end{cases} \tag{B.8}$$

The last source $u_0(k)$ corresponds to partons emitted in the beam region, which contributes only to the momentum conservation and not to the observable $K_{\rm out}$. The near-to-planar region $K_{\rm out} \ll Q$ corresponds to the region $\nu Q \gg 1$ in the Mellin variable.

Resummation can be now performed and one obtains

$$\mathcal{I}_{\tau,\delta,f}(K_{\rm out},x,\xi,Q,x_{\rm B},\eta_0) = \int \frac{d\nu\, e^{\nu K_{\rm out}}}{2\pi i\nu}\int_0^1 \frac{dx_1}{x_1}\int\frac{dN}{2\pi i}\left(\frac{x\,x_1}{x_{\rm B}}\right)^N \mathcal{P}_{\tau,f}(x_1,\mu)\,\times \\ \times \prod_{a=2,3}\int\frac{\nu d\beta_a dp_{ax}}{2\pi} e^{-\nu(|p_{ax}|-i\beta_a p_{ax})}\, e^{-\mathcal{E}_\delta}\,, \tag{B.9}$$

where $\mathcal{E}_\delta$ is the full radiator for the secondary emission and is given by

$$\mathcal{E}_\delta = \int \frac{d^3k}{\pi\omega} W_\delta(k)\;[1-U(k)\epsilon(z)]\,. \tag{B.10}$$

The 1 in the brackets here represents virtual gluon emission. The radiator $\mathcal{E}_\delta$ depends on the source variables, ν, β_a, N, η_0, on the primary parton momenta p_a, which are given in terms of Q, x, ξ and the recoil components p_{ax} (the other components can be neglected in the soft limit), and on the subtraction scale μ needed to regularize the collinear divergence. Here $W_\delta(k)$ is the two-loop distribution for the emission of the soft gluon from the three hard partons p_a in the configuration δ:

$$W_\delta(k) = \frac{N_c}{2}\left(w_{\delta a}(k)+w_{\delta b}(k)-\frac{1}{N_c^2}w_{ab}(k)\right)\,,\qquad a\neq b\neq\delta\,, \tag{B.11}$$

where $w_{ab}(k)$ is the standard distribution for emission of a soft gluon from the ab-dipole

$$w_{ab}(k) = \frac{\alpha_{\rm s}(k_{ab,t})}{\pi k^2_{ab,t}}\,,\qquad k^2_{ab,t} = \frac{2(p_ak)(kp_b)}{(p_ap_b)}\,, \tag{B.12}$$

and $\alpha_{\rm s}$ is in the physical scheme [19].

B.2 Factorization of incoming parton distribution

In the present formulation, the factorization (3.13) of the distribution $\mathcal{I}$ results by splitting the source

$$[1-U(k)\epsilon(z)] = [1-U(k)]\;+\;[1-\epsilon(z)]\,U(k)\,, \tag{B.13}$$

so that the radiator $\mathcal{E}$ can be expressed as the sum of two terms (we write explicitly only the N and μ dependence)

$$\mathcal{E}_\delta(N,\mu) = \mathcal{R}_\delta\;+\;\Gamma_\delta(N,\mu)\,, \tag{B.14}$$

with

$$\mathcal{R}_\delta = \int\frac{d^3k}{\pi\omega}\,W_\delta(k)\,[1-U(k)]\,, \\ \Gamma_\delta(N,\mu) = \int_{\mathcal{C}_1}\frac{d^3k}{\pi\omega}\,W_\delta(k)\;\left[1-z^{N-1}\right]\,U(k)\,. \tag{B.15}$$

These two terms will be evaluated for small $K_{\rm out}\sim\nu^{-1}$.

Consider first the collinear singular piece $\Gamma_\delta(N,\mu)$. Here the source $U(k)$ provides an upper bound for the integration frequencies of order $\nu^{-1} \sim K_{\rm out}$. This is shown by using, in the integral (B.15),

$$[1-u_a(k_x)] \simeq \Theta\left(|k_x|-\rho_a^{-1}\right), \qquad \rho_a = e^{\gamma_E}\nu\sqrt{1+\beta_a^2}, \tag{B.16}$$

for $a=2,3$, which is valid for large ν within SL accuracy (see for instance ref. [11, appendix C]). Using this approximate expression one also shows [12] that $\Gamma_\delta(N,\nu)$ does not depend on η_0, as long as $K_{\rm out}$ is taken in the range (2.11). As shown in [12], $\Gamma(N,\nu)$ is the soft part of the anomalous dimension that evolves the parton distributions $\mathcal{P}$ from the small subtraction scale to the scale ρ_a^{-1}. It is accurate at two loop order provided we use the coupling in the physical scheme [19], and it is diagonal in the configuration index since soft radiation is universal and does not change the nature of the incoming parton. But in general one needs the full anomalous dimension (including the hard non diagonal pieces) so that the incoming partons may change from a quark to a gluon and vice versa. The resulting exponent Γ becomes a matrix. Upon integration over x_1 and the Mellin variable N

$$\int_0^1 \frac{dx_1}{x_1}\int\frac{dN}{2\pi i}\left(\frac{x\,x_1}{x_{\rm B}}\right)^N \mathcal{P}(x_1,\mu)\cdot e^{-\Gamma(N,\mu)} \simeq \mathcal{P}\left(\frac{x_{\rm B}}{x},K_{\rm out}\right), \tag{B.17}$$

one obtains the parton distribution $\mathcal{P}(K_{\rm out})$ evolved from the subtraction to the hard scale $K_{\rm out}$, up to terms that are beyond SL.[1] The μ dependence in $\mathcal{P}(\mu)$ and $\Gamma(N,\mu)$ is cancelled. We do not consider here power corrections since they are of second order $(1/Q^2)$, see [4].

We consider then the piece $\mathcal{R}_\delta$. This is a CIS quantity ($[1-U(k)]\to 0$ for $k_t\to 0$) which produces the radiation factor $\mathcal{A}_\delta$ in (3.13)

$$\mathcal{A}_\delta(K_{\rm out},x,\xi,Q,\eta_0) = \int\frac{d\nu\, e^{\nu K_{\rm out}}}{2\pi i\nu}\,\hat{\mathcal{A}}_\delta(\nu,x,\xi,Q,\eta_0), \tag{B.18}$$

with the Mellin moment

$$\hat{\mathcal{A}}_\delta = \int\prod_{a=2,3}\frac{\nu d\beta_a dp_{ax}}{2\pi}\,e^{-\nu(|p_{ax}|-i\beta_a p_{ax})}\,e^{-\mathcal{R}_\delta}\,. \tag{B.19}$$

All the PT contributions are finite and they can be evaluated to SL accuracy by using the approximation (B.16) for the sources. The PT resummed expression $\mathcal{R}_\delta^{\rm PT}$ is then obtained by evaluating the integral (B.15) of $\mathcal{R}_\delta$ using (B.16).

However the virtual momentum k_t, entering the SL resummed running coupling in $\mathcal{R}_\delta$, cannot be prevented from going to zero. Although the contribution from this

[1] The more accurate expression $\sqrt{\mathcal{P}(\rho_2^{-1})\mathcal{P}(\rho_3^{-1})}$ is required for some of the NP contributions.

NP region is (formally) highly subleading (power suppressed), it is phenomenologically quite important for $K_{\rm out}$ not too large [3]. For k_t very small (in particular for $|k_x| \ll \nu^{-1} \sim K_{\rm out}$) the approximation (B.16) is no longer valid and we have instead

$$[1 - u_a(k)] \simeq \nu\left(|k_x| - i\beta_a k_x\right), \tag{B.20}$$

so that the leading NP correction to $\mathcal{R}_\delta$ is

$$\delta\mathcal{R}_\delta \simeq \nu \cdot \delta K_{\rm out}^{(\delta)}, \tag{B.21}$$

corresponding to a shift in $K_{\rm out}$ in (B.18). To evaluate this NP shift we use the dispersive method [4] to represent the running coupling and we evaluate the leading power-suppressed contribution from (B.20).

The two contributions to the radiator

$$\mathcal{R}_\delta = \mathcal{R}_\delta^{\rm PT} + \delta\mathcal{R}_\delta, \tag{B.22}$$

together with the expression for the radiation factor $\mathcal{A}_\delta$ will be described in the next two subsections.

B.3 PT radiation factor

The PT radiator $\mathcal{R}_\delta^{\rm PT}$ to SL accuracy coincides with a particular case of the PT radiator computed for T_m in e^+e^- annihilation (where we set $\gamma = \beta_1 = 0$ in ref. [10, eq. (4.8)]). For $K_{\rm out}$ in the range (2.11) this contribution is η_0 independent, see [12]. The explicit calculation is given in appendix C, and the result may be conveniently separated into two terms:

$$\mathcal{R}_\delta^{\rm PT} = R_{12}^{(\delta)}(\rho_2) + R_{13}^{(\delta)}(\rho_3), \tag{B.23}$$

where

$$R_{1a}^{(\delta)}(\rho_a) = \frac{1}{2}C_1^{(\delta)} r(\rho_a, \zeta_1^{(\delta)} Q_1^{(\delta)}) + C_a^{(\delta)} r(\rho_a, \zeta_a^{(\delta)} Q_a^{(\delta)}), \tag{B.24}$$

with the function r given in (3.22) and colour charges $C_a^{(\delta)}$ and hard scales $Q_a^{(\delta)}$ given in (3.21). The term $R_{12}^{(\delta)}$ corresponds to the emission in the up-region off parton p_1 and p_2. Similarly for $R_{13}^{(\delta)}$. For the PT evaluation we can expand to SL accuracy

$$\mathcal{R}_\delta^{\rm PT} = R_\delta(\nu, x, \xi, Q) + \left(C_T\gamma_E + C_{12}^{(\delta)} \ln\sqrt{1+\beta_2^2} + C_{13}^{(\delta)} \ln\sqrt{1+\beta_3^2}\right) r'(\nu, Q), \tag{B.25}$$

with R_δ the DL exponent function introduced in (3.20) and r' the SL function (3.23). Since the radiator is independent of the recoils we can integrate over p_{ax} and, using the expansion (B.25), we obtain the Mellin moment

$$\begin{aligned}\hat{\mathcal{A}}_\delta^{\rm PT}(\nu, x, \xi, Q) &= \prod_{a=2,3} \int_{-\infty}^{\infty} \frac{d\beta_a}{\pi(1+\beta_a^2)} e^{-R_{1a}^{(\delta)}} \qquad\qquad (\text{B.26})\\ &\simeq e^{-R_\delta(\nu,x,\xi,Q)}\, e^{-\gamma_E C_T r'} \mathcal{F}\left(C_{12}^{(\delta)} r'\right) \mathcal{F}\left(C_{13}^{(\delta)} r'\right), \qquad r' = r'(\nu, Q),\end{aligned}$$

with $\mathcal{F}$ given in (3.25).

The PT radiation factor is obtained by integrating over the Mellin variable ν. We make use of the operator identity

$$\int \frac{d\nu\, e^{\nu K_{\rm out}}}{2\pi i\nu}\, G(\nu) = \frac{1}{\Gamma\left(1+\frac{\partial}{\partial \ln K_{\rm out}}\right)}\, G(K_{\rm out}^{-1}) \tag{B.27}$$

for any logarithmically varying function G. (To prove this, multiply both sides by the Γ-function operator and use the definition $\Gamma(z) = \int_0^\infty dx\, x^{z-1}e^{-x}$.) Thus to SL accuracy we have

$$\mathcal{A}_\delta^{\rm PT}(K_{\rm out}, x, \xi, Q) = \frac{\hat{\mathcal{A}}_\delta^{\rm PT}(K_{\rm out}^{-1}, x, \xi, Q)}{\Gamma\left(1 + C_T r'(K_{\rm out}^{-1}, Q)\right)}, \tag{B.28}$$

which corresponds to the PT result in (3.19), (3.22) and (3.24).

B.4 Radiation factor including NP corrections

The NP correction to the radiator is computed in appendix D following the standard procedure (see [10]): we assume that the running coupling can be defined even at small momenta via a dispersion relation [4], and we take into account the non-inclusiveness of the observable via the Milan factor. The NP correction is proportional to the parameter $\lambda^{\rm NP}$, given in (D.6), expressed in terms of the integral of the running coupling in the small momentum region. This parameter takes into account merging of NP and PT corrections at two loops and is the same as entering all the jet-shape distributions so far studied. We obtain

$$\delta\mathcal{R}_\delta = \nu\lambda^{\rm NP}\left(C_1^{(\delta)} \ln \frac{x e^{\eta_0} Q_1^{(\delta)}}{Q} + \sum_{a=2,3} C_a^{(\delta)} \ln \frac{\zeta Q_a^{(\delta)}}{|p_{ax}|}\right), \qquad \zeta = 2e^{-2}, \tag{B.29}$$

where $Q_a^{(\delta)}$ are the scales introduced in (3.21). The NP radiator is made up of three contributions, one for each emitting parton. The hard scales are determined by large angle NP gluons; they are therefore the same as in the PT case. What makes the difference between the three contributions is the NP gluon rapidity cutoff. Actually, when a NP gluon is emitted from the incoming parton p_1, its rapidity is bounded by the experimental resolution η_0. This is not the case for the emission from p_2 or p_3, where it is the out-of-plane momentum of the recoiling parton which provides an effective rapidity cutoff.

The first order NP correction to the Mellin moment $\hat{\mathcal{A}}_\delta$ in (B.19) is given by

$$\hat{\mathcal{A}}_\delta(\nu) = \prod_{a=2,3} \int \frac{\nu d\beta_a dp_{3x}}{2\pi}\, e^{-\nu(|p_{ax}| - i\beta_a p_{ax})}\, e^{-R_\delta^{\rm PT}}\, \{1 - \delta\mathcal{R}_\delta\}\,, \tag{B.30}$$

where $R_\delta^{\rm PT}$ is the PT radiator in (B.23). The first term yields the PT contribution in (3.19) and so we can write

$$\hat{\mathcal{A}}_\delta(\nu) = \hat{\mathcal{A}}_\delta^{\rm PT}(\nu) - \nu\, \lambda^{\rm NP}\, f_\delta(\nu)\,. \tag{B.31}$$

After the p_{ax} integration the NP correction is given by

$$f_\delta(\nu)=\prod_{a=2,3}\int_{-\infty}^{\infty}\frac{d\beta_a}{\pi(1+\beta_a^2)}\,e^{-\mathcal{R}_\delta^{\rm PT}}\left(C_1^{(\delta)}\ln\frac{xe^{\eta_0}Q_1^{(\delta)}}{Q}+\sum_{a=2,3}C_a^{(\delta)}[\ln\zeta\,\bar\nu\,Q_a^{(\delta)}+\chi(\beta_a)]\right), \tag{B.32}$$

with $\chi(\beta)=\ln\sqrt{1+\beta^2}+\beta\tan^{-1}\beta$. This function is growing as $\pi|\beta|/2$ at large β. As a consequence, the β_a integration in (B.32) is no longer fastly convergent and we cannot expand the PT radiator as in (B.25) but we need to keep its exact expression. The region of large $|\beta_a|$, which corresponds to $|p_{ax}|\ll K_{\rm out}$, gives the leading NP correction.

The radiation factor $\mathcal{A}$ is obtained by performing the ν integration so that

$$\mathcal{A}_\delta(K_{\rm out})=\mathcal{A}_\delta^{\rm PT}(K_{\rm out})+\delta\mathcal{A}_\delta(K_{\rm out})\,. \tag{B.33}$$

In appendix E, see also [7, 10], we show that $\delta\mathcal{A}_\delta$ can be expressed (to first order) as a shift (see (3.29))

$$\delta\mathcal{A}_\delta(K_{\rm out})=-\lambda^{\rm NP}\partial_{K_{\rm out}}\int\frac{d\nu\,e^{\nu K_{\rm out}}}{2\pi i\nu}\,f_\delta(\nu)\simeq-\delta K_{\rm out}^{(\delta)}\cdot\partial_{K_{\rm out}}\mathcal{A}_\delta^{\rm PT}(K_{\rm out})\,, \tag{B.34}$$

where the shift is given by

$$\frac{\delta K_{\rm out}^{(\delta)}}{\lambda^{\rm NP}}=C_1^{(\delta)}\ln\frac{xe^{\eta_0}Q_1^{(\delta)}}{Q}+\sum_{a=2,3}C_a^{(\delta)}\left[\ln\frac{Q_a^{(\delta,{\rm NP})}}{2\bar K_{\rm out}}+\frac{C_T-C_{1a}^{(\delta)}}{C_T}\left\{E_a^{(\delta)}(\bar K_{\rm out}^{-1})+\mathcal{C}_a^{(\delta)}\right\}\right]. \tag{B.35}$$

Here the NP hard scale is defined by

$$\begin{aligned}\ln\frac{Q_a^{(\delta,{\rm NP})}}{2\bar K_{\rm out}}&=\ln\frac{\zeta Q_a^{(\delta)}}{\bar K_{\rm out}}+\psi(1+C_T\,r')+\frac{1}{2}\psi\left(1+\frac{C_{1a}^{(\delta)}\,r'}{2}\right)-\frac{1}{2}\psi\left(\frac{1+C_{1a}^{(\delta)}\,r'}{2}\right)\\&\simeq\ln\frac{2\,\zeta Q_a^{(\delta)}}{K_{\rm out}}+\mathcal{O}\left(r'\right)\,,\quad \bar K_{\rm out}=e^{-\gamma_E}K_{\rm out}\,,\end{aligned} \tag{B.36}$$

where $r'=r'(K_{\rm out}^{-1},Q)$ and the choice of factors of 2 and $e^{-\gamma_E}$ is so as to give the simple form for equation (B.42) below. The function $E_a^{(\delta)}$ is given by

$$E_a^{(\delta)}(\bar K_{\rm out}^{-1})=\int_{\bar K_{\rm out}^{-1}}^{\infty}\frac{d\rho}{\rho}e^{-R_{1a}^{(\delta)}(\rho)+R_{1a}^{(\delta)}(\bar K_{\rm out}^{-1})}\left(\frac{\mathcal{P}(\rho^{-1})}{\mathcal{P}(K_{\rm out})}\right)^{\frac{1}{2}}\,. \tag{B.37}$$

where the appearence of the parton distributions comes from the mismatch between the correct scale ρ^{-1} and the SL approximation $K_{\rm out}$ in (B.17). This introduces a dependence on τ, f and $x_{\rm B}$ which we suppress. As explained in detail in [10], these functions result from an interplay between PT and NP contributions. They can be expressed as the average of the rapidity length over the Sudakov factor for emitting in the up- or down-region for $a = 2$ or $a = 3$ respectively. Finally,

$$\mathcal{C}_a^{(\delta)} = \left(E_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) - \frac{1}{C_{1a}^{(\delta)} r'}\right)\left(\frac{1}{\mathcal{F}\left(C_{1a}^{(\delta)} r'\right)} - 1\right) \tag{B.38}$$

is a regular function of r'.

The shift can be expressed as a sum of partial shifts as in (3.30) with the rapidity integrals given by

$$\begin{aligned}
Y_{21}^{(\delta)} &= Y_{31}^{(\delta)} = \ln \frac{x e^{\eta_0} Q_1^{(\delta)}}{Q}\,, \\
Y_{22}^{(\delta)} &= \ln \frac{Q_2^{(\delta,\rm NP)}}{2\bar{K}_{\rm out}}\,, \qquad Y_{33}^{(\delta)} = \ln \frac{Q_3^{(\delta,\rm NP)}}{2\bar{K}_{\rm out}}\,, \\
Y_{23}^{(\delta)} &= L_3^{(\delta)}(\bar{K}_{\rm out}) + \mathcal{C}_3^{(\delta)}\,, \qquad Y_{32}^{(\delta)} = L_2^{(\delta)}(\bar{K}_{\rm out}) + \mathcal{C}_2^{(\delta)}\,,
\end{aligned} \tag{B.39}$$

where we have introduced the NP functions L_a defined as

$$L_a^{(\delta)}(\bar{K}_{\rm out}) = \ln \frac{Q_a^{(\delta,\rm NP)}}{2\bar{K}_{\rm out}} + E_a^{(\delta)}(\bar{K}_{\rm out}^{-1})\,. \tag{B.40}$$

These functions have the following asymptotic expansions: from (F.4) one has

$$L_a^{(\delta)}(\bar{K}_{\rm out}) = \ln \frac{Q_a^{(\delta,\rm NP)}}{2\bar{K}_{\rm out}} \left\{1 + \mathcal{O}\left(\frac{1}{\alpha_{\rm s} \ln^2 \frac{Q}{K_{\rm out}}}\right)\right\}, \qquad \alpha_{\rm s} \ln^2 \frac{Q}{K_{\rm out}} \gg 1\,, \tag{B.41}$$

while from (F.5) one obtains the behaviour of L_a in the region $\alpha_{\rm s} \ln^2 Q/K_{\rm out} \ll 1$:

$$\begin{aligned}
L_a^{(\delta)}(\bar{K}_{\rm out}) = \frac{\pi}{2\sqrt{C_{1a}^{(\delta)}\,\alpha_{\rm s}}} - \frac{1}{C_{1a}^{(\delta)}}\Bigg(&\frac{1}{2} C_1^{(\delta)} \ln \frac{Q_1^{(\delta,\rm PT)}}{Q_a^{(\delta,\rm NP)}} + C_a^{(\delta)} \ln \frac{Q_a^{(\delta,\rm PT)}}{Q_a^{(\delta,\rm NP)}} + \\
&+ \frac{\pi}{4\alpha_{\rm s}} \frac{\partial \ln \mathcal{P}(K_{\rm out})}{\partial \ln K_{\rm out}} + \frac{\beta_0}{6}\Bigg) + \mathcal{O}\left(\sqrt{\alpha_{\rm s}}\right),
\end{aligned} \tag{B.42}$$

where the PT hard scale is $Q_a^{(\delta,\rm PT)} = \zeta_a^{(\delta)} Q_a^{(\delta)}$.

C. The PT radiator

The PT radiator is given, to SL accuracy, in terms of ab-dipole radiators

$$r_{ab}(\nu, \beta_2, \beta_3) = r_{ab}^U(\nu, \beta_2) + r_{ab}^D(\nu, \beta_3)\,, \tag{C.1}$$

where the 'up' and 'down' hemisphere components are given by

$$r_{ab}^{U}(\nu,\beta_2) = \int_{k_y>0} \frac{d^3k}{\pi\omega} w_{ab}(k)\,\left[1-e^{-\nu(|k_x|-i\beta_2 k_x)}\right], \qquad w_{ab}(k) = \frac{\alpha_s(k_{ab,t}^2)}{\pi k_{ab,t}^2},$$
$$r_{ab}^{D}(\nu,\beta_3) = \int_{k_y<0} \frac{d^3k}{\pi\omega} w_{ab}(k)\,\left[1-e^{-\nu(|k_x|-i\beta_3 k_x)}\right], \tag{C.2}$$

and $k_{ab,t}$ is the invariant transverse momentum of k with respect to the P_a, P_b hard partons in (A.1). For the configuration $\delta = 1$, for instance, we have

$$R_1^{\mathrm{PT}}(\nu,\beta_a) = \frac{N_c}{2}\left(r_{12}(\nu,\beta_a) + r_{13}(\nu,\beta_a) - \frac{1}{N_c^2}\,r_{23}(\nu,\beta_a)\right). \tag{C.3}$$

To evaluate the ab-dipole radiator $r_{ab}(\nu,\beta_a)$ we work in the centre of mass system of the ab-dipole. We neglect the rapidity cut (2.3): the correction is beyond our accuracy [12]. Denoting by P_a^*, P_b^* and k^* the momenta in this system, we introduce the Sudakov decomposition

$$P_a^* = \frac{Q_{ab}}{2}(1,0,0,1)\,, \quad P_b^* = \frac{Q_{ab}}{2}(1,0,0,-1)\,, \qquad k^* = \alpha P_a^* + \beta P_b^* + \kappa\,, \tag{C.4}$$

where the scales $Q_{ab}^2 = 2(P_aP_b)$ are given in (3.3). Here the two-dimensional vector $\vec{\kappa}$ is the transverse momentum orthogonal to the ab-dipole momenta ($\kappa^2 = k_{ab,t}^2$). We have then

$$w_{ab}(k) = \frac{\alpha_s(\kappa^2)}{\pi\kappa^2}\,, \qquad \frac{d^3k}{\pi\omega} = \frac{d^2\kappa}{\pi}\frac{d\alpha}{\alpha}\,, \quad \alpha > \frac{\kappa^2}{Q_{ab}^2}\,. \tag{C.5}$$

Since, neglecting the recoils, the outgoing momenta P_2 and P_3 are in the yz-plane, the Lorentz transformation is in the yz-plane and our observable k_x remains unchanged. However, the boundary between the 'up' and 'down' hemispheres becomes non-trivial, and different for each dipole, so we must take each in turn.

The 12-radiator with

$$k^* = \alpha P_2^* + \beta P_1^* + \kappa\,, \tag{C.6}$$

has up and down hemispheres given by

$$U\;:\kappa_y > -\frac{QT_M}{2}\alpha\,, \qquad D\;:\kappa_y < -\frac{QT_M}{2}\alpha\,, \tag{C.7}$$

with T_M related to ξ by (A.2). The up component is then

$$r_{12}^{U}(\nu,\beta_2) = \int_0^{Q_{12}^2} \frac{d^2\kappa}{\pi\kappa^2}\frac{\alpha_s(\kappa)}{\pi}\int_{\kappa^2/Q_{12}^2}^{1}\frac{d\alpha}{\alpha}\left[1-e^{-\nu(|\kappa_x|-i\beta_2\kappa_x)}\right]\Theta\left(\alpha+\frac{2\kappa_y}{QT_M}\right). \tag{C.8}$$

Integrating over α and κ_y we have

$$r_{12}^U(\nu,\beta_2) = \int_{-Q_{12}}^{Q_{12}} \frac{dk_x}{|k_x|}\left[1-e^{-\nu(|k_x|-i\beta_2 k_x)}\right]\cdot\frac{\alpha_s(2|k_x|)}{\pi}\left(\ln\frac{Q_{12}}{2|k_x|}+\frac{1}{2}\ln\frac{QT_M}{2|k_x|}\right). \tag{C.9}$$

To show this we introduced $\kappa_y = t\cdot|k_x|$ and used

$$\int_{-\infty}^{\infty}\frac{dt}{\pi(1+t^2)}\alpha_s\left(|k_x|\sqrt{1+t^2}\right)\ln\frac{Q_{ab}}{|k_x|\sqrt{1+t^2}} \simeq \alpha_s(2|k_x|)\ln\frac{Q_{ab}}{2|k_x|}\,. \tag{C.10}$$

We extended the t-integration to infinity since it is convergent, then we integrated over t by expanding α_s to second order. Corrections are beyond SL accuracy.

Finally, using (B.16), we obtain

$$r_{12}^U(\nu,\beta_2) = 2\int_{\rho_2^{-1}}^{Q_{12}}\frac{dk_x}{k_x}\frac{\alpha_s(2k_x)}{\pi}\left(\ln\frac{Q_{12}}{2k_x}+\frac{1}{2}\ln\frac{QT_M}{2k_x}\right). \tag{C.11}$$

Similarly the 'down' hemisphere contribution is

$$r_{12}^D(\nu,\beta_3) = 2\int_{\rho_3^{-1}}^{Q_{12}}\frac{dk_x}{k_x}\frac{\alpha_s(2k_x)}{\pi}\left(\ln\frac{Q_{12}}{2k_x}-\frac{1}{2}\ln\frac{QT_M}{2k_x}\right). \tag{C.12}$$

The 13 radiator has analogous results

$$\begin{aligned} r_{13}^U(\nu,\beta_2) &= 2\int_{\rho_2^{-1}}^{Q_{13}}\frac{dk_x}{k_x}\frac{\alpha_s(2k_x)}{\pi}\left(\ln\frac{Q_{13}}{2k_x}-\frac{1}{2}\ln\frac{QT_M}{2k_x}\right), \\ r_{13}^D(\nu,\beta_3) &= 2\int_{\rho_3^{-1}}^{Q_{13}}\frac{dk_x}{k_x}\frac{\alpha_s(2k_x)}{\pi}\left(\ln\frac{Q_{13}}{2k_x}+\frac{1}{2}\ln\frac{QT_M}{2k_x}\right). \end{aligned} \tag{C.13}$$

The 23-radiator has a slightly different structure: with

$$k^* = \alpha P_2^* + \beta P_3^* + \kappa\,, \tag{C.14}$$

the up and down hemispheres are given by

$$U\ :\kappa_y > \frac{QT_M}{2}\frac{\beta-\alpha}{1-2\xi}\,,\qquad D\ :\kappa_y < \frac{QT_M}{2}\frac{\beta-\alpha}{1-2\xi}\,. \tag{C.15}$$

The up component is then

$$\begin{aligned} r_{23}^U(\nu,\beta_2) = &\int_0^{Q_{23}^2}\frac{d^2\kappa}{\pi\kappa^2}\frac{\alpha_s(\kappa)}{\pi}\int_{\kappa^2/Q_{23}^2}^1\frac{d\alpha}{\alpha}\left[1-e^{-\nu(|\kappa_x|-i\beta_2\kappa_x)}\right]\times \\ &\times\Theta\left(\alpha-\frac{\kappa_y(1-2\xi)}{QT_M}-\sqrt{\frac{\kappa_y^2}{Q^2T_M^2}+\frac{\kappa_x^2}{Q_{23}^2}}\right), \end{aligned} \tag{C.16}$$

Performing the integrals as above we obtain

$$r_{23}^{U}(\nu,\beta_2) = 2\int_{\rho_2^{-1}}^{Q_{23}} \frac{dk_x}{k_x}\frac{\alpha_{\rm s}(2k_x)}{\pi}\ln\frac{Q_{23}}{2k_x}\,, \tag{C.17}$$

and similarly

$$r_{23}^{D}(\nu,\beta_3) = 2\int_{\rho_3^{-1}}^{Q_{23}} \frac{dk_x}{k_x}\frac{\alpha_{\rm s}(2k_x)}{\pi}\ln\frac{Q_{23}}{2k_x}\,. \tag{C.18}$$

Assembling the various dipole contributions and including hard collinear splittings then yields, to SL accuracy,

$$\begin{aligned} R_\delta^{\rm PT}(\nu,\beta_2,\beta_3) = {}& \frac{1}{2}\sum_{a=1}^{3} C_a^{(\delta)}\left[r(\rho_2,\zeta_a^{(\delta)}Q_a^{(\delta)}) + r(\rho_3,\zeta_a^{(\delta)}Q_a^{(\delta)})\right] + \\ & +\frac{1}{2}\left(C_2^{(\delta)} - C_3^{(\delta)}\right)\left[r(\rho_2,QT_M) - r(\rho_3,QT_M)\right]\,. \end{aligned} \tag{C.19}$$

Finally, we note that the terms on the second line contribute only at SL level, since the DL terms cancel. We may therefore change at will the hard scales in these terms, yielding the result (B.23).

D. NP corrections to the radiator

We consider the NP correction δr_{ab} to the ab-dipole radiator. In this case, as we shall see, we need to retain both the recoil momenta p_{2x} and p_{3x} and the rapidity cut η_0. We write the integral in the ab-dipole centre of mass variables α, β and $\vec{\kappa}$ introduced in (C.4) and, to obtain the NP correction δr_{ab}, we perform the following standard operations:

- the running coupling, reconstructed by two loop emission, is represented by the dispersive form [4]. Then, the ab-dipole radiation $w_{ab}(k)$ is written in the ab-centre of mass system (see (C.5)) in the form

$$w_{ab}(k) = \frac{\alpha_{\rm s}(\kappa)}{\pi\kappa^2} = \int_0^\infty \frac{dm^2\,\alpha_{\rm eff}(m)}{\pi(\kappa^2+m^2)^2}\,; \tag{D.1}$$

- to take into account the emission of soft partons at two loop order [6], we need to extend the source $u(k_x)$ to include the mass m of the soft system. We assume $k_x = \kappa\cos\phi \to \sqrt{\kappa^2+m^2}\cos\phi$, with ϕ the azimuthal angle of $\vec{\kappa}$. Similarly we introduce the mass in the kinematical relations such as $\alpha\beta = (\kappa^2+m^2)/Q_{ab}^2$ for the ab-dipole variables;

- we take the NP part $\delta\alpha_{\rm eff}(m)$ of the effective coupling. Since it has support only for small m, we take the leading part of the integrand for small κ, and m. In particular we linearize the source $U(k)$

$$[1-U(k)] \;\to\; \nu\sqrt{\kappa^2+m^2}\,|\cos\phi|\,\Theta(\eta_0-\eta_k)\,. \tag{D.2}$$

  Recall that η_k is the rapidity of k in the Breit frame (2.1). Here we have neglected terms proportional to β_2 and β_3 since they vanish, by symmetry, upon the β_a integration;

- the recoil component p_{ax} of the outgoing parton p_a does provide an effective cut in the soft gluon rapidity along the outgoing parton [10, 7]. This is due to a real-virtual cancellation which takes place when the angle of the outgoing parton p_a with the event plane exceeds the corresponding angle of the soft gluon. The detailed analysis of real and virtual pieces entails that the contribution from the observable $\sqrt{\kappa^2+m^2}|\cos\phi|$ in the linear expansion of the source (see (D.2)) has to be replaced by

$$\sqrt{\kappa^2+m^2}\,|\cos\phi| \to \left|\sqrt{\kappa^2+m^2}\,\cos\phi+\alpha p_{ax}\right|-\alpha|p_{ax}|\,, \tag{D.3}$$

  with α the Sudakov variable in the ab-dipole centre of mass in the forward region $\alpha>\beta$;

- to take fully into account effects of non-inclusiveness of jet observables at two-loop order, we multiply the radiator by the Milan factor [6, 16]

$$\mathcal{M}=\frac{3}{64}\frac{(128\pi+128\pi\ln 2-35\pi^2)C_A-5\pi^2 n_f}{11C_A-2n_f}\,, \tag{D.4}$$

  using $n_f=3$;

- the NP correction is finally expressed in terms of the parameter

$$\lambda^{\rm NP}=2\mathcal{M}\,c_{K_{\rm out}}\int dm\frac{\delta\alpha_{\rm eff}(m)}{\pi}\,,\quad c_{K_{\rm out}}=\frac{2}{\pi}\,. \tag{D.5}$$

After merging PT and NP contributions to the observable in a renormalon free manner, one has that the distribution is independent of μ_I and one obtains

$$\lambda^{\rm NP}\equiv c_{K_{\rm out}}\,\mathcal{M}\frac{4}{\pi^2}\mu_I\left\{\alpha_0(\mu_I)-\bar\alpha_s-\beta_0\frac{\bar\alpha_s^2}{2\pi}\left(\ln\frac{Q}{\mu_I}+\frac{K}{\beta_0}+1\right)\right\}, \tag{D.6}$$

where

$$\bar\alpha_s\equiv\alpha_{\overline{\rm MS}}(Q)\,,\quad K\equiv C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{5}{9}n_f\,,\quad \beta_0=\frac{11N_c}{3}-\frac{2n_f}{3}\,. \tag{D.7}$$

The K factor accounts for the mismatch between the $\overline{\text{MS}}$ and the physical scheme [19] and $\alpha_0(\mu_I)$ is the integral of the running coupling over the infrared region, see (3.31).

The numerical coefficient $c_{K_{\text{out}}}$ depends on our observable K_{out}. For instance, the shift for the $\tau = 1-T$ distribution is

$$\frac{d\sigma}{d\tau}(\tau) = \frac{d\sigma^{\text{PT}}}{d\tau}(\tau-\Delta_\tau)\,, \qquad \Delta_\tau = C_F\,\frac{c_\tau\,\lambda^{\text{NP}}}{c_{K_{\text{out}}}}\,, \quad c_\tau = 2\,, \tag{D.8}$$

where C_F enters due to the fact the 2-jet system is made of a quark-antiquark pair.

We recall that these prescriptions correspond to taking into account NP corrections at two-loop order in the reconstruction of the (dispersive) running coupling and in the non-inclusive nature of the observable. We implement the rapidity cut by expressing η_k the soft gluon rapidity in the invariant form (2.2).

D.1 Dipoles 12 and 13

Consider first the contribution of the $1a$-dipole ($a = 2, 3$). We decompose the gluon momenta along P_1 and P_a taken in their centre of mass system:

$$P_a^* = \frac{Q_{1a}}{2}(1,0,0,1)\,, \quad P_1^* = \frac{Q_{1a}}{2}(1,0,0,-1)\,, \quad k^* = \alpha P_a^* + \beta P_1^* + \kappa\,. \tag{D.9}$$

and we use the expression in (D.3) in our linearized source, so that the recoil momentum of parton p_a provides an effective rapidity cutoff for a gluon emitted collinear to P_a. In the region in which k^* is emitted close to P_1, i.e. $\beta \gg \alpha$, we can write gluon rapidity in terms of the Sudakov variables (D.9) as follows:

$$\eta_k = \frac{1}{2}\ln\frac{(x_{\text{B}}P+q)k}{x_{\text{B}}(Pk)} \simeq \ln\frac{Q\sqrt{\kappa^2+m^2}}{\alpha Q_{1a}^2 x}\,, \tag{D.10}$$

so that the NP correction to the $1a$-dipole radiator is given by

$$\begin{aligned}
\delta r_{1a} &= \frac{\nu\mathcal{M}}{\pi}\int dm^2\delta\alpha_{\text{eff}}(m)\frac{-d}{dm^2}\int\frac{d\kappa^2}{\kappa^2+m^2}\,I_{1a}\,, \\
I_{1a} &= \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\int_0^1\frac{d\alpha}{\alpha}\left(\left|\sqrt{\kappa^2+m^2}\cos\phi+\alpha p_{ax}\right|-\alpha|p_{ax}|\right)\Theta\left(\eta_0+\ln\frac{\alpha Q_{1a}^2\,x}{Q\sqrt{\kappa^2+m^2}}\right).
\end{aligned} \tag{D.11}$$

If we consider the two rapidity cutoffs discussed above we have that the α integration is restricted to the region

$$\alpha_m < \alpha < \alpha_M\,, \qquad \alpha_m \equiv \frac{\sqrt{\kappa^2+m^2}\,Q\,e^{-\eta_0}}{Q_{1a}^2 x}\,, \quad \alpha_M \equiv \frac{\sqrt{\kappa^2+m^2}}{|p_{ax}|}\,, \tag{D.12}$$

giving

$$I_{1a} = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \int_{\alpha_m}^{\alpha_M} \frac{d\alpha}{\alpha} \left(\left| \sqrt{\kappa^2+m^2} \cos\phi + \alpha p_{ax} \right| - \alpha |p_{ax}| \right)$$
$$= \frac{2}{\pi} \sqrt{\kappa^2 + m^2} \left(\eta_0 + \ln \frac{Q_{1a} x}{Q} + \ln \frac{Q_{1a} \zeta}{|p_{ax}|} \right), \qquad \zeta = 2e^{-2}. \tag{D.13}$$

We thus obtain the NP correction to the $1a$-dipole radiator

$$\delta r_{1a} = \nu \lambda^{\rm NP} \left(\eta_0 + \ln \frac{Q_{1a} x}{Q} + \ln \frac{Q_{1a} \zeta}{|p_{ax}|} \right). \tag{D.14}$$

D.2 Dipole 23

Again we decompose the emitted gluon momentum along P_2 and P_3

$$P_2^* = \frac{Q_{23}}{2}(1,0,0,1), \quad P_3^* = \frac{Q_{23}}{2}(1,0,0,-1), \quad k^* = \alpha P_2^* + \beta P_3^* + \kappa. \tag{D.15}$$

In this case, if k^* is emitted close to P_2^* ($\alpha > \beta$), its rapidity is cut by the recoil component p_{2x}, while it is p_{3x} which provides an effective rapidity cutoff when k^* is close to P_3^* ($\alpha < \beta$). Thus it is convenient to split the radiator into forward ($\alpha > \beta$) and backward ($\alpha < \beta$) regions. In the backward region one may relabel β as α. Performing the substitution in (D.3) in the linearized source we obtain

$$\delta r_{23} = \frac{\nu \mathcal{M}}{\pi} \int dm^2 \delta\alpha_{\rm eff}(m) \frac{-d}{dm^2} \int \frac{d\kappa^2}{\kappa^2 + m^2} \left(I_{23}^2 + I_{23}^3 \right), \tag{D.16}$$
$$I_{23}^a = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \int_0^1 \frac{d\alpha}{\alpha} \left(\left| \sqrt{\kappa^2+m^2} \cos\phi + \alpha p_{ax} \right| - \alpha |p_{ax}| \right) \Theta \left(\alpha - \frac{\sqrt{\kappa^2+m^2}}{Q_{23}} \right),$$

(for $a = 2, 3$), so that the integration limits for α become

$$\alpha_m < \alpha < \alpha_M, \qquad \alpha_m \equiv \frac{\sqrt{\kappa^2+m^2}}{Q_{23}}, \quad \alpha_M \equiv \frac{\sqrt{\kappa^2+m^2}}{|p_{ax}|}, \tag{D.17}$$

giving the following result

$$I_{23}^a = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \int_{\alpha_m}^{\alpha_M} \frac{d\alpha}{\alpha} \left(\left| \sqrt{\kappa^2+m^2} \cos\phi + \alpha p_{ax} \right| - \alpha |p_{ax}| \right)$$
$$= \frac{2}{\pi} \sqrt{\kappa^2 + m^2} \left(\ln \frac{Q_{23} \zeta}{|p_{ax}|} \right), \qquad \zeta = 2\, e^{-2}. \tag{D.18}$$

Putting together the two pieces one obtains the NP correction to r_{23}

$$\delta r_{23} = \nu \lambda^{\rm NP} \left(\ln \frac{Q_{23} \zeta}{|p_{2x}|} + \ln \frac{Q_{23} \zeta}{|p_{3x}|} \right). \tag{D.19}$$

In conclusion, assembling the contributions from the three dipoles, one is left with the expression in (B.29).

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E. Evaluating the NP shift

Here we evaluate the NP shifts $\delta K_{\rm out}^{(\delta)}$ introduced in (B.34). Using the operator identity (B.27) we may write

$$\delta\mathcal{A}_\delta(K_{\rm out}) = \frac{-\lambda^{\rm NP}}{K_{\rm out}} \frac{\frac{\partial}{\partial \ln K_{\rm out}}}{\Gamma\left(1+\frac{\partial}{\partial\ln K_{\rm out}}\right)}\, f_\delta(K_{\rm out}^{-1})\,. \tag{E.1}$$

Applying this differential operator yields

$$\begin{aligned}\delta\mathcal{A}_\delta(K_{\rm out}) = {} & \frac{-\lambda^{\rm NP}}{K_{\rm out}} \frac{1}{\Gamma(1+R')} \prod_{a=2,3} \int_{-\infty}^{\infty} \frac{d\beta_a\, e^{-R_{1a}^{(\delta)}\left(\bar{K}_{\rm out}^{-1}\sqrt{1+\beta_a^2}\right)}}{\pi(1+\beta_a^2)} \times \\ & \times \Bigg[C_1^{(\delta)} R' \ln\frac{x e^{\eta_0} Q_1^{(\delta)}}{Q} + \sum_{a=2,3} \times \\ & \qquad \times C_a^{(\delta)} \left\{ R' \ln(\zeta \bar{K}_{\rm out}^{-1} Q_a^{(\delta)}) + R'\psi(1+R'_T) + R'\chi(\beta_a) - 1 \right\} \Bigg]\,, \end{aligned} \tag{E.2}$$

with $\bar{K}_{\rm out} = K_{\rm out} e^{-\gamma_E}$, $R' = C_T r'(K_{\rm out}^{-1}, Q)$ and $R_{1a}^{(\delta)}$ defined in (B.24). The final term may now be integrated by parts to give

$$\begin{aligned}\delta\mathcal{A}_\delta(K_{\rm out}) = {} & \frac{-\lambda^{\rm NP}}{K_{\rm out}} \frac{R'}{\Gamma(1+R')} \prod_{a=2,3} \int_{-\infty}^{\infty} \frac{d\beta_a\, e^{-R_{1a}^{(\delta)}\left(\bar{K}_{\rm out}^{-1}\sqrt{1+\beta_a^2}\right)}}{\pi(1+\beta_a^2)} \times \\ & \times \Bigg[C_1^{(\delta)} \ln\frac{x e^{\eta_0} Q_1^{(\delta)}}{Q} + \sum_{a=2,3} C_a^{(\delta)} \Bigg\{ \ln(\zeta \bar{K}_{\rm out}^{-1} Q_a^{(\delta)}) + \psi(1+R') + \\ & \qquad + \frac{1}{2}\ln(1+\beta_a^2) + \left(1 - \frac{C_{1a}^{(\delta)}}{C_T}\right) \beta_a \tan^{-1}\beta_a \Bigg\} \Bigg]\,. \end{aligned} \tag{E.3}$$

For all but the final term in the brackets, we may expand the radiator as in the PT calculation and perform the integrals over β_a. The final term is only slowly convergent and must be treated with extra care. Thus

$$\delta\mathcal{A}_\delta(K_{\rm out}) = -\delta K_{\rm out}^{(\delta)} \cdot \frac{R'}{K_{\rm out}} \mathcal{A}_\delta^{\rm PT}(K_{\rm out}) = -\delta K_{\rm out}^{(\delta)} \cdot \partial_{K_{\rm out}} \mathcal{A}_\delta^{\rm PT} \tag{E.4}$$

with

$$\begin{aligned}\frac{\delta K_{\rm out}^{(\delta)}}{\lambda^{\rm NP}} = {} & C_1^{(\delta)} \ln\frac{x e^{\eta_0} Q_1^{(\delta)}}{Q} + \sum_{a=2,3} C_a^{(\delta)} \Bigg[\ln\frac{\zeta Q_a^{(\delta)}}{\bar{K}_{\rm out}} + \psi(1+C_T r') + \frac{1}{2}\psi\left(1+\frac{C_{1a}^{(\delta)} r'}{2}\right) - \\ & - \frac{1}{2}\psi\left(\frac{1+C_{1a}^{(\delta)} r'}{2}\right) + H_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) \Bigg]\,. \end{aligned} \tag{E.5}$$

The term called $H_a^{(\delta)}(\bar{K}_{\rm out}^{-1})$ arises from this final term. For the term proportional to $C_2^{(\delta)}$ we may expand the radiator in β_3 and integrate over it, and vice versa, giving

$$H_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) = \left(1 - \frac{C_{1a}^{(\delta)}}{C_T}\right) \frac{e^{R_{1a}^{(\delta)}(\bar{K}_{\rm out}^{-1})}}{\mathcal{F}(C_{1a}^{(\delta)} r')} \int_{-\infty}^{\infty} \frac{d\beta_a \, e^{-R_{1a}^{(\delta)}\left(\bar{K}_{\rm out}^{-1}\sqrt{1+\beta_a^2}\right)}}{\pi(1+\beta_a^2)} \beta_a \tan^{-1} \beta_a \,. \quad \text{(E.6)}$$

The reason we may not simply expand the remaining radiator and integrate over β_a is that doing so gives an unphysical result that diverges in the limit $r'(K_{\rm out}^{-1}) \to 0$. This unphysical divergence is regulated by the second derivative of the radiator. We write

$$\beta \tan^{-1} \beta = \frac{\pi}{2}|\beta| - \beta \tan^{-1} \frac{1}{\beta} \,. \quad \text{(E.7)}$$

By making the substitution $\rho = \bar{K}_{\rm out}^{-1}\sqrt{1+\beta_a^2}$, the contribution from the first term is written in terms of the function E_a, discussed in the next appendix. The second contribution gives a fastly convergent integral, so we can expand the radiator and integrate as before. We find

$$H_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) = \left(1 - \frac{C_{1a}^{(\delta)}}{C_T}\right) \frac{1}{\mathcal{F}(C_{1a}^{(\delta)} r')} \left(E_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) + \frac{\mathcal{F}(C_{1a}^{(\delta)} r') - 1}{C_{1a}^{(\delta)} r'}\right) . \quad \text{(E.8)}$$

where the E_a functions are discussed below. Thus we recover the result (B.35).

F. The E_a functions

For the functions $E_a^{(\delta)}(\bar{K}_{\rm out}^{-1})$ we obtain the following result:

$$\begin{aligned} E_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) &\equiv \int_{\bar{K}_{\rm out}^{-1}}^{\infty} \frac{d\rho}{\rho} \, e^{-R_{1a}^{(\delta)}(\rho) + R_{1a}^{(\delta)}(\bar{K}_{\rm out}^{-1})} \left(\frac{\mathcal{P}(\rho^{-1})}{\mathcal{P}(K_{\rm out})}\right)^{1/2} \\ &= \sqrt{\frac{\pi}{2R''_{1a}}} \, N(t) - \frac{R'''_{1a}}{3R''^{2}_{1a}} \, X(t) + \mathcal{O}\left(\sqrt{\alpha_{\rm s}}\right) , \\ & \qquad t \equiv \frac{1}{\sqrt{2R''_{1a}}} \left(R'_{1a} + \frac{1}{2} \frac{\partial \ln \mathcal{P}(K_{\rm out})}{\partial \ln K_{\rm out}}\right) , \end{aligned} \quad \text{(F.1)}$$

where R'_{1a}, R''_{1a} and R'''_{1a} are the first three logarithmic derivatives of $R_{1a}^{(\delta)}(\rho)$ evaluated at $\rho = \bar{K}_{\rm out}^{-1}$, and the functions N and X are defined by

$$N(t) \equiv \frac{2}{\sqrt{\pi}} \int_t^{\infty} dx \, e^{-x^2+t^2} \,, \qquad X(t) \equiv -\frac{\sqrt{\pi}}{8} \frac{d^3 N(t)}{dt^3} = \int_0^{\infty} dz \, z \, e^{-z-2t\sqrt{z}} . \quad \text{(F.2)}$$

They have the following asymptotic behaviours

$$t \gg 1: \quad N(t) = \frac{1}{\sqrt{\pi} t}\left(1 - \frac{1}{2t^2} + \frac{3}{t^4} + \cdots\right), \quad X(t) = \frac{3}{4t^4} + \mathcal{O}\left(t^{-6}\right) , \quad \text{(F.3)}$$

$$t \ll 1: \quad N(t) = 1 - \frac{2t}{\sqrt{\pi}} + t^2 - \frac{4t^3}{3\sqrt{\pi}} + \cdots , \quad X(t) = 1 - \frac{3\sqrt{\pi}}{2} t + 4t^2 + \mathcal{O}\left(t^3\right) .$$

In the region $R'_{1a}/\sqrt{2R''_{1a}} \gg 1$, using the expansions in (F.3), we get

$$E_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) = \frac{1}{R'_{1a}}\left\{1+\mathcal{O}\left(\frac{1}{R'_{1a}}\frac{\partial \ln \mathcal{P}(K_{\rm out})}{\partial \ln K_{\rm out}}\right)\right\} = \frac{1}{C_{1a}^{(\delta)} r'}\left\{1+\mathcal{O}\left(\frac{1}{\ln(Q/K_{\rm out})}\right)\right\}. \tag{F.4}$$

On the contrary, for $R'_{1a}/\sqrt{2R''_{1a}} \ll 1$, we obtain, up to contributions $\mathcal{O}\left(\sqrt{\alpha_{\rm s}}\right)$,

$$E_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) = \sqrt{\frac{\pi}{2R''_{1a}}} - \frac{R'_{1a}}{R''_{1a}} - \frac{1}{2R''_{1a}}\frac{\partial \ln \mathcal{P}(K_{\rm out})}{\partial \ln K_{\rm out}} - \frac{R'''_{1a}}{3R''^2_{1a}} + \cdots . \tag{F.5}$$

Starting from the definition (B.24) we find

$$\begin{aligned}
R'_{1a} &= \frac{2}{\pi}\alpha_{\rm s}(2\bar{K}_{\rm out})\left(\frac{1}{2}C_1^{(\delta)}\ln\frac{\zeta_1^{(\delta)}Q_1^{(\delta)}}{2\bar{K}_{\rm out}} + C_a \ln\frac{\zeta_a^{(\delta)}Q_a^{(\delta)}}{2\bar{K}_{\rm out}}\right)\\
R''_{1a} &= \frac{2}{\pi}C_{1a}^{(\delta)}\alpha_{\rm s}(2\bar{K}_{\rm out}) + \frac{\beta_0}{\pi^2}\alpha_{\rm s}^2(2\bar{K}_{\rm out})\left(\frac{1}{2}C_1^{(\delta)}\ln\frac{\zeta_1^{(\delta)}Q_1^{(\delta)}}{2\bar{K}_{\rm out}} + C_a \ln\frac{\zeta_a^{(\delta)}Q_a^{(\delta)}}{2\bar{K}_{\rm out}}\right)\\
R'''_{1a} &= \frac{2\beta_0}{\pi^2}C_{1a}^{(\delta)}\alpha_{\rm s}^2 + \cdots
\end{aligned} \tag{F.6}$$

and therefore

$$\begin{aligned}
E_a^{(\delta)}(\bar{K}_{\rm out}^{-1}) = \frac{\pi}{2\sqrt{C_{1a}^{(\delta)}\alpha_{\rm s}}} - \frac{1}{C_{1a}^{(\delta)}}\Bigg(&\frac{1}{2}C_1^{(\delta)}\ln\frac{\zeta_1^{(\delta)}Q_1^{(\delta)}}{2\bar{K}_{\rm out}} + C_a \ln\frac{\zeta_a^{(\delta)}Q_a^{(\delta)}}{2\bar{K}_{\rm out}} + \\
&+ \frac{\pi}{4\alpha_{\rm s}}\frac{\partial \ln \mathcal{P}(K_{\rm out})}{\partial \ln K_{\rm out}} + \frac{\beta_0}{6}\Bigg) + \mathcal{O}\left(\sqrt{\alpha_{\rm s}}\right) .
\end{aligned} \tag{F.7}$$

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